

1

Pacing Guide

Chapter 1: Essentials of Geometry

Chapter Table of Contents

- 1.1** Identify Points, Lines, and Planes
- 1.2** Use Segments and Congruence
- 1.3** Use Midpoint and Distance Formulas
- 1.4** Measure and Classify Angles

Investigating Geometry Construction:

Copy and Bisect Segments
and Angles

- 1.5** Describe Angle Pair Relationships
- 1.6** Classify Polygons

PACING GUIDES

Regular Schedule (50-minute classes)

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	DAY 6
Lesson 1.1	Lesson 1.2	Lesson 1.3	Lesson 1.3 (cont.) Mixed Review of Problem Solving	Quiz Lesson 1.4	Lesson 1.4 (cont.) Investigating Geometry Construction
DAY 7	DAY 8	DAY 9	DAY 10	DAY 11	DAY 12
Lesson 1.5	Lesson 1.6 Mixed Review of Problem Solving	Quiz Chapter Review	Chapter Test	Standardized Test Preparation and Practice	

Block Schedule (90-minute classes)

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	DAY 6
Lesson 1.1 Lesson 1.2	Lesson 1.3 Mixed Review of Problem Solving	Quiz Lesson 1.4 Investigating Geometry Construction	Lesson 1.5 Lesson 1.6	Quiz Mixed Review of Problem Solving Chapter Review	Chapter Test Standardized Test Preparation and Practice

RESOURCE OPTIONS

Chapter/Lesson Resources

Chapter Resource Book

- Parents as Partners
- Teaching Guide/Lesson Plan
- Activity Masters
- Practice (3 levels)
- Study Guide
- Quick Catch-Up for Absent Students
- Problem Solving/Application
- Challenge Practice
- Chapter Review Games and Activities
- Project with Rubric
- Cumulative Review

Notetaking Guide

- Student Workbook and Teacher's Edition

Practice Workbook

Worked-Out Solution Key

Chapter Transparencies, Online only

- Warm-Up Exercises/Daily Homework Quiz
- Notetaking Guide Transparencies
- Homework Answer Transparencies

Teacher Tools Transparencies

Assessment

Assessment Book

- Quizzes
- Chapter Tests (3 levels)
- Standardized and SAT/ACT Chapter Tests
- Alternative Assessments
- Cumulative Tests

Benchmark Tests

- Benchmark Tests, correlated to Remediation Book
- Pre-Course, Mid-Year, and End-of-Year Tests
- Chapter Tests

Spanish Assessment Book

Differentiated Instruction

Differentiated Instruction Resources

- Strategies for Reading Mathematics
- Differentiated Instruction Lesson Notes
- English Learner Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Tips for New Teachers/Math Background Notes
- Teacher Survival Activities/Bulletin Board Ideas

Student Resources in Spanish

Spanish Study Guide

Remediation Book

Skills Readiness (available online)

- Diagnostic Assessment
- Skill Instruction and Alternative Teaching Strategies
- Skill Practice and Enrichment Masters

Pre-AP Resources

- Pacing and Assignment Guide
- Best Practices
- Copymasters



Technology Resources

Plan	Lesson Plans (available online)
Teach	Activity Generator Power Presentations Animated Geometry
Assess	ExamView™ Assessment Suite Online Assessment
Reteach	@HomeTutor
Online Resources	my.hrw.com eEdition



Technology Highlights for Each Lesson

Teacher One-Stop

Easy access to chapter resources and assessments. Includes lesson planning, test generation, and puzzle creation software.



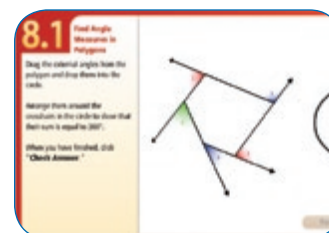
Activity Generator

Leveled, editable activities allow all students to explore a lesson's concepts. Includes teacher notes and closure questions.



Animated Geometry

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.

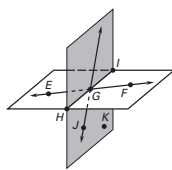


LESSON 1.1 Practice B

For use with the lesson "Identify Points, Lines, and Planes"

Use the diagram to decide whether the given statement is true or false.

- Points H , I , and G are collinear.
- Points H , I , and J are coplanar.
- \overleftrightarrow{EG} and \overleftrightarrow{FG} are opposite rays.
- All points on \overleftrightarrow{GI} and \overleftrightarrow{GF} are coplanar.
- The intersection of \overleftrightarrow{EF} and plane JKH is \overleftrightarrow{HI} .
- The intersection of \overleftrightarrow{EF} , \overleftrightarrow{HI} , and \overleftrightarrow{JG} is point G .
- The intersection of plane EGH and plane JGI is point G .
- The intersection of plane EFI and plane JKG is \overleftrightarrow{HG} .

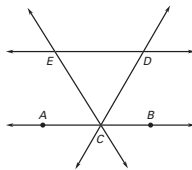


Sketch the figure described.

- Two rays that do not intersect
- Three planes that intersect in one line
- Three lines that intersect in three points
- A ray that intersects a plane in one point

In Exercises 13–15, use the diagram.

- Name 12 different rays.
- Name a pair of opposite rays.
- Name 3 lines that intersect at point C .



- Draw four noncollinear points A , B , C , and D . Then sketch \overleftrightarrow{AB} , \overleftrightarrow{BC} , and \overleftrightarrow{AD} .
- Sketch plane M intersecting plane N . Then sketch plane O so that it intersects plane N , but not plane M .

LESSON 1.1 Practice B continued

For use with the lesson "Identify Points, Lines, and Planes"

You are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

- $y = 5x + 3$; $A(1, 8)$
- $y = -x + 3$; $A(6, 3)$
- $y = -3x - 6$; $A(2, 0)$
- $2x - y = 7$; $A(3, -1)$
- $x + 6y = 40$; $A(-10, 5)$
- $-x - 4y = -14$; $A(-6, 2)$

Graph the inequality on a number line. Tell whether the graph is a segment, a ray or rays, a point, or a line.

- $x \geq 2$
- $2 \leq x \leq 5$



- $x \leq 0$ or $x \geq 8$



- $|x| \leq 0$



28. Counter Stools Two different types of stools are shown below.

- One stool rocks slightly from side to side on your kitchen floor. Which of the two stools could this possibly be? Explain why this might occur.
- Suppose that each stool is placed on a flat surface that is slightly sloped. Do you expect either of the stools to rock from side to side? Explain why or why not.



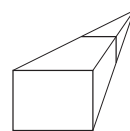
Three-legged stool



Four-legged stool

29. Perspective Drawings Recall from the text, that a perspective drawing is drawn using vanishing points.

- Does the drawing at the right represent a perspective drawing? Explain why or why not.
- Using heavy dashed lines, draw the hidden lines of the prism.
- Redraw the prism so that it uses two vanishing points.



LESSON 1.2 Practice B

For use with the lesson "Use Segments and Congruence"

Measure the length of the segment to the nearest tenth of a centimeter.

- \overline{AB}
- \overline{MN}
- \overline{EF}

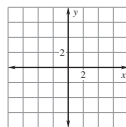
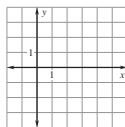
Use the Segment Addition Postulate to find the indicated length.

- Find RT .
- Find BC .
- Find MN .

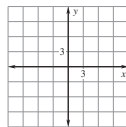
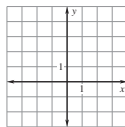


Plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

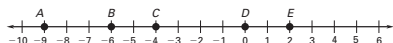
- $A(2, 2)$, $B(4, 2)$, $C(-1, -1)$, $D(-1, 1)$; \overline{AB} and \overline{CD}
- $M(1, -3)$, $N(4, -3)$, $O(3, 4)$, $P(4, 4)$; \overline{MN} and \overline{OP}



- $E(-3, 4)$, $F(-1, 4)$, $G(2, 4)$, $H(-1, 1)$; \overline{EG} and \overline{FH}
- $R(3, 5)$, $S(10, 5)$, $T(-4, -3)$, $U(-11, -3)$; \overline{RS} and \overline{TU}



Use the number line to find the indicated distance.



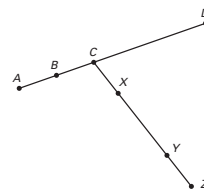
- AB
- AD
- CD
- BD
- CE
- AE
- BE
- DE

LESSON 1.2 Practice B continued

For use with the lesson "Use Segments and Congruence"

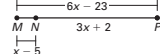
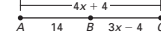
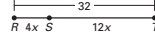
In the diagram, points A , B , C , and D are collinear, points C , X , Y , and Z are collinear, $AB = BC = CX = YZ$, $AD = 54$, $XY = 22$, and $XZ = 33$. Find the indicated length.

- AB
- BD
- CY
- CD
- XC
- CZ

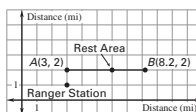


Find the indicated length.

- Find ST .
- Find AC .
- Find NP .

Point J is between H and K on \overline{HK} . Use the given information to write an equation in terms of x . Solve the equation. Then find HJ and JK .

- $HJ = 2x$
 $JK = 3x$
 $KH = 25$
- $HJ = 5x - 4$
 $JK = 8x - 10$
 $KH = 38$
- $HJ = \frac{x}{4}$
 $JK = 3x - 4$
 $KH = 22$
- $HJ = 5x - 3$
 $JK = x - 9$
 $KH = 5x$

32. Hiking On the map, \overline{AB} represents a trail that you are hiking. You start from the beginning of the trail and hike for 90 minutes at a rate of 1.4 miles per hour. How much farther do you need to hike to reach the end of the trail?

LESSON 1.3 Practice B For use with the lesson "Use Midpoint and Distance Formulas"

- Line RS bisects \overline{PQ} at point R . Find RQ if $PQ = 14$ centimeters.
- Line JK bisects \overline{MN} at point J . Find MN if $JM = 6\frac{3}{4}$ feet.
- Point T bisects \overline{UV} . Find UV if $UT = 4\frac{1}{2}$ yards.
- Point C bisects \overline{AB} . Find CB if $AB = 14.8$ meters.

In the diagram, M is the midpoint of the segment. Find the indicated length.

- Find LN .
- Find AM .
- Find MR .

Find the coordinates of the midpoint of the segment with the given endpoints.

- $S(4, -1)$ and $T(6, 0)$
- $L(4, 2)$ and $P(0, 2)$
- $H(-5, 5)$ and $I(7, 3)$
- $G(-2, -8)$ and $H(-3, -12)$

Use the given endpoint R and midpoint M of \overline{RS} to find the coordinates of the other endpoint S .

- $R(6, 0)$, $M(0, 2)$
- $R(3, 4)$, $M(3, -2)$
- $R(-3, -2)$, $M(-1, -8)$
- $R(11, -5)$, $M(-4, -4)$

Find the length of the segment. Round to the nearest tenth of a unit.

-
-
-
-

Geometry
Chapter Resource Book 1-37

LESSON 1.3 Practice B *continued* For use with the lesson "Use Midpoint and Distance Formulas"

Find the length of the segment. Then find the coordinates of the midpoint of the segment.

-
-

The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.

- \overline{AB} : $A(2, 6)$, $B(0, 3)$
 \overline{CD} : $C(-1, 0)$, $D(1, 3)$
- \overline{RS} : $R(5, 4)$, $S(0, 4)$
 \overline{TU} : $T(-4, -3)$, $U(-1, 1)$
- \overline{KL} : $K(-4, 13)$, $L(-10, 6)$
 \overline{MN} : $M(-1, -2)$, $N(-1, -11)$
- \overline{OP} : $O(6, -2)$, $P(3, -2)$
 \overline{QR} : $Q(5, 2)$, $R(1, 5)$

- Distances** Your house and the mall are 9.6 miles apart on the same straight road. The movie theater is halfway between your house and the mall, on the same road.
 - Draw and label a sketch to represent this situation. How far is your house from the movie theater?
 - You walk at an average speed of 3.2 miles per hour. About how long would it take you to walk to the movie theater from your house?

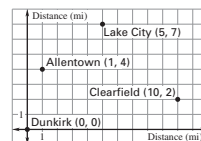
In Exercises 27–29, use the map. The locations of the towns on the map are: Dunkirk (0, 0), Clearfield (10, 2), Lake City (5, 7), and Allentown (1, 4). The coordinates are given in miles.

- Find the distance between each pair of towns. Round to the nearest tenth of a mile.

- Which two towns are closest together? Which two towns are farthest apart?

- The map is being used to plan a 26-mile marathon. Which of the following plans is the best route for the marathon? *Explain.*

- Dunkirk to Clearfield to Allentown to Dunkirk
- Dunkirk to Clearfield to Lake City to Allentown to Dunkirk
- Dunkirk to Lake City to Clearfield to Dunkirk
- Dunkirk to Lake City to Allentown to Dunkirk



LESSON 1.3

Geometry
Chapter Resource Book 1-38

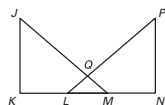
LESSON 1.4 Practice B For use with the lesson "Measure and Classify Angles"

Use a protractor to measure the angle to the nearest degree. Write two names for the angle. Then name the vertex and the sides of the angle.

-
-
-

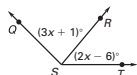
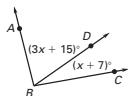
Give another name for the angle in the diagram. Tell whether the angle appears to be acute, obtuse, right, or straight.

- $\angle JKN$
- $\angle KMN$
- $\angle PQM$
- $\angle JML$
- $\angle QPN$
- $\angle PLK$



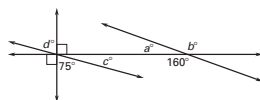
Use the given information to find the indicated angle measure.

- Given $m\angle ABC = 94^\circ$, find $m\angle CBD$.
- Given $m\angle QST = 135^\circ$, find $m\angle QSR$.



Find the indicated angle measure.

- a°
- b°
- c°
- d°



In the diagram, \overline{BD} bisects $\angle ABC$. Find $m\angle ABC$.

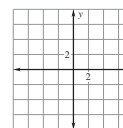
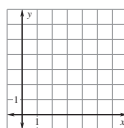
-
-
-

Geometry
Chapter Resource Book 1-52

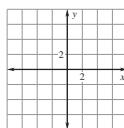
LESSON 1.4 Practice B *continued* For use with the lesson "Measure and Classify Angles"

Plot the points in a coordinate plane and draw $\angle ABC$. Classify the angle. Then give the coordinates of a point that lies in the interior of the angle.

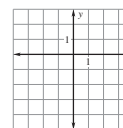
- $A(2, 3)$, $B(3, 0)$, $C(2, 6)$
- $A(6, 2)$, $B(-1, -2)$, $C(2, 3)$



- $A(-4, -3)$, $B(-1, 3)$, $C(4, 4)$

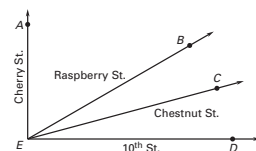


- $A(-2, -4)$, $B(-2, -1)$, $C(3, -1)$

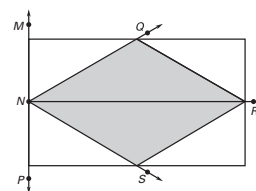


- Let $(3x + 24)^\circ$ represent the measure of an obtuse angle. What are the possible values of x ?

- Streets** The diagram shows the intersection of four streets. In the diagram, $m\angle AEB = 60^\circ$, $m\angle BEC = m\angle CED$, and $\angle AED$ is a right angle. What is the measure of $\angle CED$?



- Flags** In the flag shown, $\angle MNP$ is a straight angle and \overline{NR} bisects $\angle MNP$ and $\angle QNS$. Use only the labeled angles in the diagram.
 - Which angles are acute? obtuse? right?
 - Identify the congruent angles.
 - If $m\angle QNR = 30^\circ$, find $m\angle MNR$, $m\angle RNS$, $m\angle QNS$, and $m\angle QNP$.



Geometry
Chapter Resource Book 1-53

LESSON
1.5

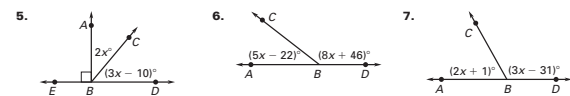
Practice B

For use with the lesson "Describe Angle Pair Relationships"

$\angle 1$ and $\angle 2$ are complementary angles and $\angle 2$ and $\angle 3$ are supplementary angles. Given the measure of $\angle 1$, find $m\angle 2$ and $m\angle 3$.

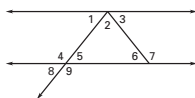
1. $m\angle 1 = 80^\circ$ 2. $m\angle 1 = 33^\circ$ 3. $m\angle 1 = 72^\circ$ 4. $m\angle 1 = 7^\circ$

Find $m\angle ABC$ and $m\angle CBD$.



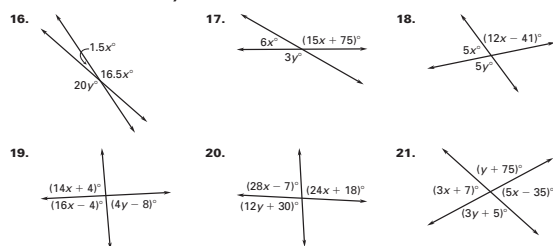
In Exercises 8–12, use the diagram. Tell whether the angles are vertical angles, a linear pair, or neither.

8. $\angle 1$ and $\angle 3$
9. $\angle 2$ and $\angle 3$
10. $\angle 4$ and $\angle 5$
11. $\angle 5$ and $\angle 8$
12. $\angle 4$ and $\angle 9$



13. The measure of one angle is three times the measure of its complement. Find the measure of each angle.
14. Two angles form a linear pair. The measure of one angle is 8 times the measure of the other angle. Find the measure of each angle.
15. The measure of one angle is 38° less than the measure of its supplement. Find the measure of each angle.

Find the values of x and y .



Geometry
Chapter Resource Book 1-65

LESSON
1.5

Practice B

continued
For use with the lesson "Describe Angle Pair Relationships"

Tell whether the statement is *always*, *sometimes*, or *never* true.

22. Two complementary angles form a linear pair.
23. The supplement of an obtuse angle is an acute angle.
24. An angle that has a supplement also has a complement.

$\angle A$ and $\angle B$ are complementary angles. Find $m\angle A$ and $m\angle B$.

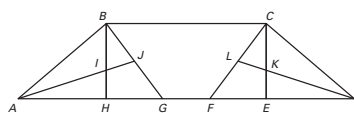
25. $m\angle A = x^\circ$ 26. $m\angle A = (4x + 34)^\circ$
 $m\angle B = (2x - 75)^\circ$ $m\angle B = (x + 36)^\circ$
27. $m\angle A = (4x - 18)^\circ$ 28. $m\angle A = (2x + 10)^\circ$
 $m\angle B = (6x - 18)^\circ$ $m\angle B = (-x + 55)^\circ$

$\angle A$ and $\angle B$ are supplementary angles. Find $m\angle A$ and $m\angle B$.

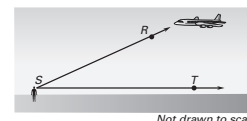
29. $m\angle A = (x + 50)^\circ$ 30. $m\angle A = 6x^\circ$
 $m\angle B = (x + 100)^\circ$ $m\angle B = (x + 5)^\circ$
31. $m\angle A = (2x + 3)^\circ$ 32. $m\angle A = (-4x + 40)^\circ$
 $m\angle B = (3x - 223)^\circ$ $m\angle B = (x + 50)^\circ$

Roof trusses can have several different layouts. The diagram below shows one type of roof truss made out of beams of wood. Use the diagram to identify two different examples of the indicated type of angle pair. In the diagram, $\angle HBC$ and $\angle BCE$ are right angles.

33. Supplementary angles
34. Complementary angles
35. Vertical angles
36. Linear pair angles
37. Adjacent angles



38. **Angle of elevation** An angle of elevation is the angle between the horizontal line and the line of sight of an object above the horizontal. In the diagram, a plane is flying horizontally across the sky and $\angle RST$ represents the angle of elevation. How is the angle of elevation affected as the plane flies closer to the person? Explain.



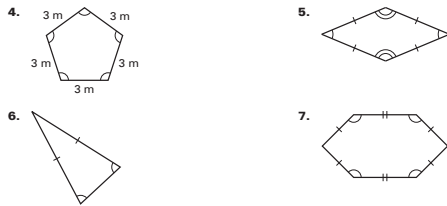
Not drawn to scale

Geometry
1-66 Chapter Resource Book

Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is *convex* or *concave*.



Classify the polygon by the number of sides. Tell whether the polygon is *equilateral*, *equiangular*, or *regular*. Explain your reasoning.



8. The lengths (in feet) of two sides of a regular quadrilateral are represented by the expressions $8x - 6$ and $4x + 22$. Find the length of a side of the quadrilateral.
9. The expressions $(3x + 63)^\circ$ and $(7x - 45)^\circ$ represent the measures of two angles of a regular decagon. Find the measure of an angle of the decagon.
10. The expressions $-2x + 41$ and $7x - 40$ represent the lengths (in kilometers) of two sides of an equilateral pentagon. Find the length of a side of the pentagon.

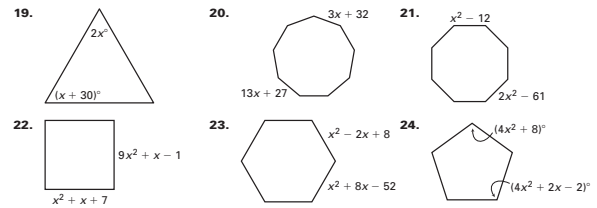
Tell whether the statement is *always*, *sometimes*, or *never* true.

11. A quadrilateral is convex.
12. An octagon is regular.
13. A triangle is concave.
14. A regular polygon is equilateral.

Draw a figure that fits the description.

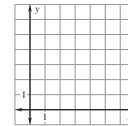
15. A quadrilateral that is not regular
16. A convex heptagon
17. A concave pentagon
18. An equiangular hexagon that is not equilateral

Each figure is a regular polygon. Find the value of x .

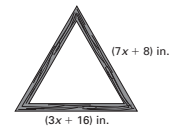


25. The vertices of a figure are given below. Plot and connect the points so that they form a convex polygon. Classify the figure. Then show that the figure is equilateral using algebra.

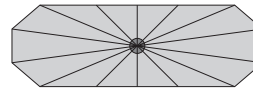
$A(3, 0)$, $B(3, 6)$, $C(2, 3)$, $D(4, 3)$



26. **Picture frames** A picture frame with a wooden border is a regular triangle, as shown. You want to decorate the frame by wrapping a ribbon around it. How many feet of ribbon are needed to wrap the ribbon around the border one time?



27. **Parachutes** The canopy of a parachute is shown in the diagram.



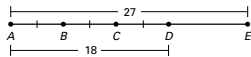
- a. Is the shape of the canopy a *convex* or *concave* polygon?
- b. Classify the polygon by the number of sides. Then use a ruler and a protractor to determine whether the figure is equilateral, equiangular, or regular.
- c. Determine the number of lines of symmetry in the canopy. How does this differ from a regular octagon?

CHAPTER 1 Quiz 1

For use after the lessons "Identify Points, Lines, and Planes", "Use Segments and Congruence", and "Use Midpoint and Distance Formulas"

1. Sketch a plane and a line that intersects the plane at a point.

Find the indicated length.



2. DE 3. AB
4. AC 5. BD
6. CE 7. BE

8. The endpoints of \overline{ST} are $S(-3, 2)$ and $T(5, 8)$. Find the coordinates of the midpoint of \overline{ST} . Then find ST .

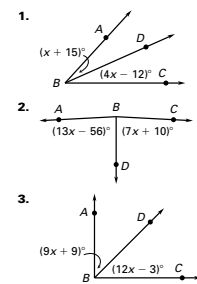
Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

CHAPTER 1 Quiz 2

For use after the lessons "Measure and Classify Angles", "Describe Angle Pair Relationships", and "Classify Polygons"

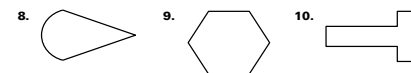
In the diagram, \overline{BD} bisects $\angle ABC$. Find $m\angle ABD$ and $m\angle DBC$.



Find the measure of the complement of $\angle 1$ and the measure of the supplement of $\angle 1$.

4. $m\angle 1 = 51^\circ$ 5. $m\angle 1 = 17^\circ$
6. $m\angle 1 = 80^\circ$ 7. $m\angle 1 = 3^\circ$

Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is *convex* or *concave*.



Answers

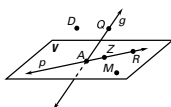
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

CHAPTER 1 Chapter Test B

For use after the chapter "Essentials of Geometry"

In Exercises 1–3, use the diagram to decide whether the statement is *true* or *false*.

1. Point R lies on line g .
2. Points A , M , R , and Z are coplanar.
3. Points A and Q are collinear.

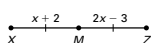


4. The diagram shows three houses on a street. Find the distance from House A to House C .

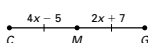


In each diagram, M is the midpoint of the segment. Find the indicated length.

5. XM

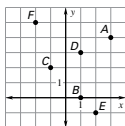


6. CG



Find the exact distance between the points.

7. A and B
8. C and F
9. D and E



Use the endpoint and midpoint M of the segment to find the coordinates of the other endpoint.

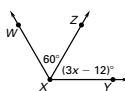
10. \overline{AB} ; $A(-1, 3)$, $M(3, 1)$ 11. \overline{CD} ; $C(-2, -3)$, $M(-4, 2)$

- Use the given information to find the value of x .

12. $\angle ABD \cong \angle DBC$



13. $\angle WXZ \cong \angle ZXY$



Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____

CHAPTER 1 Chapter Test B

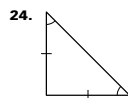
continued
For use after the chapter "Essentials of Geometry"

14. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 17^\circ$, find $m\angle 1$.
15. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 46^\circ$, find $m\angle 4$.
16. Two angles form a linear pair. The measure of one angle is four times greater than the measure of the other angle. Find the measure of each angle.
17. Two angles form a linear pair. The measure of one angle is six more than twice the measure of the other angle. Find the measure of each angle.

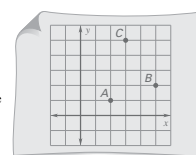
Tell whether the statement is *always*, *sometimes*, or *never* true.

18. A pentagon is a plane figure.
19. A triangle is concave.
20. A hexagon has six congruent sides.
21. A quadrilateral is equiangular but not equilateral.
22. The complement of an acute angle is an acute angle.
23. The supplement of an acute angle is an obtuse angle.

Classify the polygon by the number of sides. Tell whether the polygon is *equilateral*, *equiangular*, or *regular*. Explain your reasoning.



24. The lengths of two sides of a regular hexagon are $3x - 8$ and $2x - 1$. Find the length of each side of the hexagon.
25. The expressions $(5x + 5)^\circ$ and $(4x + 13)^\circ$ represent the measures of two angles of a regular octagon. Find the measure of an angle of the octagon.
26. Joe ran from Point A to Point C and Mike ran from Point B to Point C . About how much farther did Joe run than Mike? Round your answer to the nearest tenth. The distance between consecutive grid lines represents 1 yard.



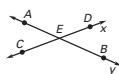
Answers

14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____

CHAPTER 1 Standardized Test For use after the chapter "Essentials of Geometry"

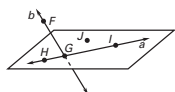
Multiple Choice

1. Which statement about the figure is true?



- (A) Lines x and y intersect at point A .
(B) Points A , B , and C are collinear.
(C) \overline{EC} and \overline{ED} are opposite rays.
(D) Another name for \overline{AE} is \overline{AB} .

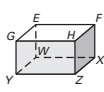
2. Name three points that are collinear.



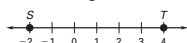
- (A) G , H , and I (B) H , G , and J
(C) F , G , and I (D) G , J , and I

3. What is the intersection of plane HGY and plane HFV ?

- (A) \overline{HZ}
(B) \overline{HJ}
(C) Point H
(D) Plane EFH

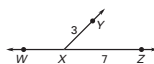


4. What is the length of \overline{ST} ?



- (A) 2 (B) 4 (C) -2 (D) 6

5. If $\overline{WX} \cong \overline{XY}$, what is the length of \overline{WZ} ?



- (A) 7 (B) 10 (C) 3 (D) 4

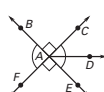
6. The endpoints of \overline{CD} are $C(6, 1)$ and $D(-4, -1)$. Find the midpoint M of \overline{CD} .

- (A) $M(10, 2)$ (B) $M(-10, -2)$
(C) $M(2, 0)$ (D) $M(1, 0)$

7. \overline{JK} has a length of 4.5 units. If \overline{LM} has endpoints $L(3, 1)$ and $M(-1, 4)$, how much longer than \overline{JK} is \overline{LM} ?

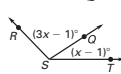
- (A) 0.5 unit (B) 2 units
(C) 2.5 units (D) \overline{JK} is longer.

8. Name the acute angles in the given figure.



- (A) $\angle CAD$ and $\angle DAE$
(B) $\angle BAC$ and $\angle FAE$
(C) $\angle BAF$ and $\angle CAE$
(D) $\angle BAD$ and $\angle FAD$

9. If the measure of $\angle RST$ is 134° , find the measure of $\angle QST$.



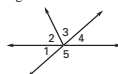
- (A) 67° (B) 33° (C) 34° (D) 98°

10. $m\angle A$ is 42° greater than $m\angle B$. If $\angle A$ and $\angle B$ are supplementary, find $m\angle A$ and $m\angle B$.

- (A) $m\angle A = 111^\circ$, $m\angle B = 69^\circ$
(B) $m\angle A = 42^\circ$, $m\angle B = 48^\circ$
(C) $m\angle A = 42^\circ$, $m\angle B = 138^\circ$
(D) $m\angle A = 66^\circ$, $m\angle B = 24^\circ$

CHAPTER 1 Standardized Test For use after the chapter "Essentials of Geometry"

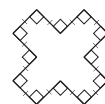
11. Name a pair of vertical angles in the figure shown.



- (A) $\angle 2$ and $\angle 4$ (B) $\angle 1$ and $\angle 4$
(C) $\angle 3$ and $\angle 5$ (D) There are none.

12. Which describes the following polygon?

- (A) equilateral
(B) equiangular
(C) regular
(D) none of these



13. Which of the following is a convex polygon?

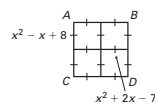
- (A) (B)
(C) (D)

14. Point M is the midpoint of \overline{AB} . If $AM = 12x + 8$ and $MB = 10x + 15$, find the length of \overline{AB} .

- (A) 3.5 (B) 4
(C) 50 (D) 100

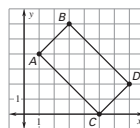
15. Find CD .

- (A) 5
(B) 28
(C) 56
(D) 96



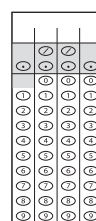
16. Find the length of diagonal \overline{BC} of $ABCD$ to the nearest hundredth.

- (A) 2.83 units
(B) 5.66 units
(C) 6.32 units
(D) 7.21 units



Gridded Answer

17. Find the area, in square units, of a triangle with vertices $X(-7, 2)$, $Y(8, 2)$, and $Z(6, 7)$.



Short Response

18. A swimmer stands somewhere in a circular pool. The distance to the farthest side (through the center of the pool) is 3 times the distance to the nearest side. The circumference of the pool is 100 feet.

- a. How close is the swimmer to the nearest side?
b. How far must the swimmer swim to get to the center?

Extended Response

19. You are a surveyor. You take your first measurement facing due north. You turn to the right to take your second measurement and then right again, 4 times as far, to take your third measurement. You are now facing due west.

- a. How many degrees did you turn to take your second measurement?
b. How many degrees should you have turned after your second measurement if you wanted to take your third measurement facing south?
c. How many degrees must you turn to the left in order to take a fourth measurement in the opposite direction of your second measurement?

CHAPTER 1 Alternative Assessment and Math Journal

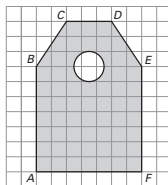
For use after the chapter "Essentials of Geometry"

- Journal** 1. a. If three points are collinear, are the three points necessarily coplanar? Explain. Use examples to support your reasoning.

- b. Are all equiangular polygons equilateral polygons? Are all equilateral polygons equiangular polygons? Explain. Use examples to support your reasoning.

Multi-Step Problem

2. A company that makes craft supplies is designing a metal tag that is used in crafting projects such as making scrapbooks. The tag is shown below.



- a. The tag is a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral*, *equiangular*, or *regular*.
b. Draw axes on the figure above and find the coordinates of the vertices of the tag.
c. The center of the tag's hole is located midway between point B and point E . What are the coordinates of the center of the hole?
d. Identify any congruent line segments formed by the tag.
e. Given that $m\angle ERA = 90^\circ$ and $m\angle CBA = 150^\circ$, find $m\angle CBE$.
f. Find the approximate length of the diagonal from B to F .

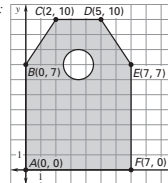
CHAPTER 1 Alternative Assessment Rubric For use after the chapter "Essentials of Geometry"

Journal Solution

1. Complete answers should include:
a. an explanation that three collinear points must be coplanar; example of three collinear points that are coplanar.
b. explanations that all equiangular polygons are not equilateral and all equilateral polygons are not equiangular; examples of an equiangular polygon that is not equilateral (e.g. rectangle) and an equilateral polygon that is not equiangular (e.g. rhombus).

Multi-Step Problem Solution

2. a. hexagon; not equilateral, equiangular, or regular
b. Sample answer:



- c. Sample answer: (3.5, 7)
d. \overline{AB} , \overline{AF} , and \overline{EF} ; \overline{CB} and \overline{DE}
e. $m\angle CBE = 60^\circ$
f. about 9.9 units

Multi-Step Problem Rubric

- 4 The student answers all parts of the problem correctly and completely. The student shows all work. The student's work is neat.
3 The student answers all parts of the problem. The student's work may contain one or two errors in the calculations of the circle's center, area or circumference, or the tag perimeter. The student shows most work. The student's work is neat.
2 The student answers all parts of the problem. The student's work contains more than two errors in the calculations. The student shows some work. The student's work is sloppy.
1 The student does not complete all parts of the problem. The work contains many errors in the calculations. The student's work is sloppy, or no work is shown.

PLAN AND PREPARE

Main Ideas

In this chapter students will name and sketch geometric figures, use postulates to identify congruent segments, find lengths of segments in the coordinate plane, and find the midpoint of a segment. Students also will name, measure and classify angles, identify complementary and supplementary angles, and classify polygons.

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from <i>Skills Readiness</i>
Any of Exs. 3–6 answered incorrectly	Skill 54 Simplify absolute value expressions
Any of Exs. 7–10 answered incorrectly	Skill 60 Evaluate variable expressions
Any of Exs. 11–16 answered incorrectly	Skill 69 Solve equations
All exercises answered correctly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

1

Essentials of Geometry

- 1.1 Identify Points, Lines, and Planes
- 1.2 Use Segments and Congruence
- 1.3 Use Midpoint and Distance Formulas
- 1.4 Measure and Classify Angles
- 1.5 Describe Angle Pair Relationships
- 1.6 Classify Polygons

Before

Previously, you learned the following skills, which you'll use in this chapter: finding measures, simplifying and evaluating expressions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

- The distance around a polygon is called its ? , and the distance around a circle is called its ? . **perimeter, circumference**
- The number of square units covered by a figure is called its ? . **area**

SKILLS AND ALGEBRA CHECK

Simplify the expression.

3. $|4 - 6|$ **2** 4. $|3 - 11|$ **8** 5. $|-4 + 5|$ **1** 6. $|-8 - 10|$ **18**

Evaluate the expression when $x = 2$.

7. $5x$ **10** 8. $20 - 8x$ **4** 9. $-18 + 3x$ **-12** 10. $-5x - 4 + 2x$ **-10**

Solve the equation.

11. $274 = -2z$ **-137** 12. $8x + 12 = 60$ **6** 13. $2y - 5 + 7y = -32$ **-3**
14. $6p + 11 + 3p = -7$ **-2** 15. $4 + \frac{m}{7} = 10$ **42** 16. $5n - 8 = 47$ **11**

Chapter Planning Guide

Chapter Resource Book

- Teaching Guide/Lesson Plan
- Project with Rubric

Assessment and Intervention

- Assessment Book
- Benchmark Tests
- Remediation Book
- Skills Readiness

Interactive Technology

- Power Presentations
- Activity Generator
- Animated Geometry
- ExamView™ Assessment Suite
- Online Quizzes
- eEdition
- @HomeTutor

Resources for English Learners

- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

- 1 Describing geometric figures
- 2 Measuring geometric figures
- 3 Understanding equality and congruence

KEY VOCABULARY

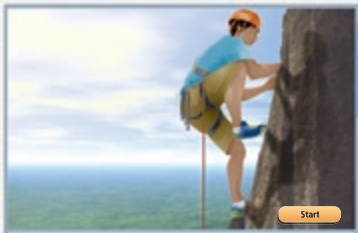
- undefined terms
point, line, plane
- defined terms
- line segment, endpoints
- ray, opposite rays
- postulate, axiom
- congruent segments
- midpoint
- segment bisector
- acute, right, obtuse, straight angles
- congruent angles
- angle bisector
- linear pair
- vertical angles
- polygon
- convex, concave
- n -gon
- equilateral, equiangular, regular

Why?

Geometric figures can be used to represent real-world situations. For example, you can show a climber's position along a stretched rope by a point on a line segment.

Animated Geometry

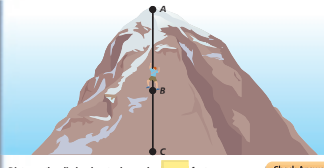
The animation illustrated below helps you answer a question from this chapter: How far must a climber descend to reach the bottom of a cliff?



Your goal is to find the distance from a climber's position to the bottom of a cliff.

Start

AC is 52 feet and AB is 31 feet. How much farther must the climber descend to reach the bottom of the cliff? Enter your answer in the box below and click "Check Answer."



Distance the climber has to descend = feet.

Check Answer

Use the given information to enter a distance. Then check your answer.

Animated Geometry at my.hrw.com

Differentiated Instruction Resources

- Reading Strategies
- Differentiated Instruction Lesson Notes
- English Learners Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Using Technology in the Classroom
- Tips for New Teachers
- Math Background Notes
- Assessment Strategies
- Teacher Survival Activities
- Bulletin Board Idea

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Graph each inequality.

1. $x \leq 1$



2. $-2 \leq x \leq 3$



3. Juan has more than 5 but fewer than 11 fish in his aquarium.

Write an inequality to express the number of fish f Juan has.

$5 < f < 11$

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with next lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you name geometric figures? **Tell students they will learn how to answer this question by learning about labeling points, lines, segments, rays, and planes.**

1.1 Identify Points, Lines, and Planes

Before

You studied basic concepts of geometry.

Now

You will name and sketch geometric figures.

Why

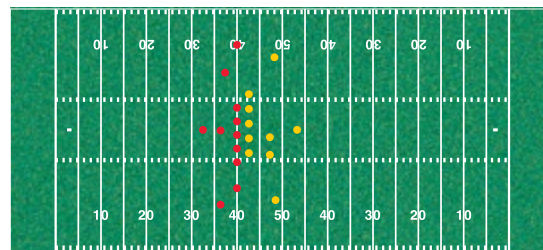
So you can use geometry terms in the real world, as in Ex. 13.



Key Vocabulary

- **undefined terms**
point, line, plane
- **collinear points**
- **coplanar points**
- **defined terms**
- **line segment**
- **endpoints**
- **ray**
- **opposite rays**
- **intersection**

In the diagram of a football field, the positions of players are represented by *points*. The yard lines suggest *lines*, and the flat surface of the playing field can be thought of as a *plane*.



In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

KEY CONCEPT

For Your Notebook

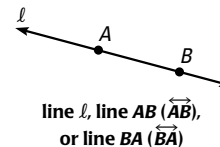
Undefined Terms

Point A **point** has no dimension. It is represented by a dot.



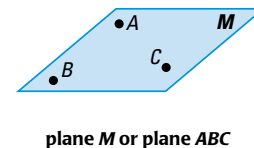
Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

Through any two points, there is exactly one line. You can use any two points on a line to name it.



Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Name points, lines, and planes

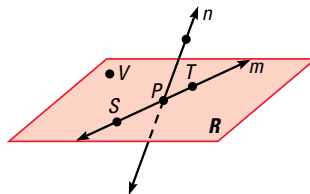
VISUAL REASONING

There is a line through points S and Q that is not shown in the diagram. Try to imagine what plane SPQ would look like if it were shown.

- Give two other names for \overleftrightarrow{PQ} and for plane R .
- Name three points that are collinear. Name four points that are coplanar.

Solution

- Other names for \overleftrightarrow{PQ} are \overleftrightarrow{QP} and line n . Other names for plane R are plane SVT and plane PTV .
- Points S , P , and T lie on the same line, so they are collinear. Points S , P , T , and V lie in the same plane, so they are coplanar.



Animated Geometry at my.hrw.com



GUIDED PRACTICE for Example 1

- Use the diagram in Example 1. Give two other names for \overleftrightarrow{ST} . Name a point that is *not* coplanar with points Q , S , and T . **Sample answer:** \overleftrightarrow{TS} , \overleftrightarrow{PT} ; point V

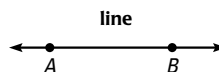
DEFINED TERMS In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

KEY CONCEPT

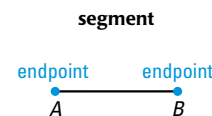
For Your Notebook

Defined Terms: Segments and Rays

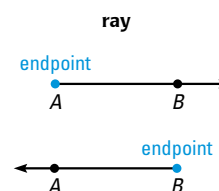
Line AB (written as \overleftrightarrow{AB}) and points A and B are used here to define the terms below.



Segment The **line segment** AB , or **segment** AB , (written as \overline{AB}) consists of the **endpoints** A and B and all points on \overleftrightarrow{AB} that are between A and B . Note that \overline{AB} can also be named \overline{BA} .

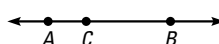


Ray The **ray** AB (written as \overrightarrow{AB}) consists of the endpoint A and all points on \overleftrightarrow{AB} that lie on the same side of A as B .



Note that \overrightarrow{AB} and \overrightarrow{BA} are different rays.

If point C lies on \overleftrightarrow{AB} between A and B , then \overrightarrow{CA} and \overrightarrow{CB} are **opposite rays**.



Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.

Motivating the Lesson

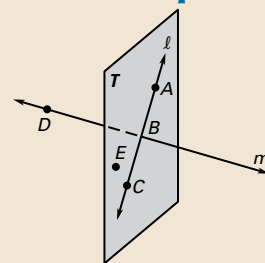
Ask students to identify a point, line, segment, and flat surface in the classroom. Help them describe the intersection of one wall with the ceiling as a segment, and the intersection of two walls and the ceiling as a point.

3 TEACH

Teaching Strategy

Students can benefit from a discussion of some real world examples of the ideas in the Key Concepts box on page 2. The ideas that two points determine a line and that three non-collinear points determine a plane will be revisited and formally presented as postulates in the next chapter.

Extra Example 1

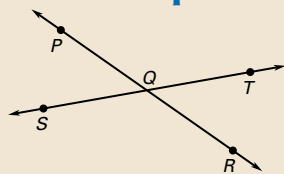


- Give two other names for \overleftrightarrow{BD} . **DB, m**
- Give another name for plane T . **Sample answer: plane ABE**
- Name three points that are collinear. **A, B, C**
- Name four points that are coplanar. **A, B, C, E**

Animated Geometry
my.hrw.com

An **Animated Geometry** activity is available online for **Example 1**. This activity is also part of **Power Presentations**.

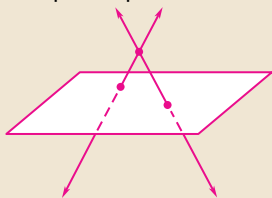
Extra Example 2



- Give another name for \overrightarrow{PR} . \overrightarrow{RP}
- Name all rays with endpoint Q . Which of these rays are opposite rays? \overrightarrow{QP} , \overrightarrow{QR} , \overrightarrow{QT} , \overrightarrow{QS} , \overrightarrow{QT} and \overrightarrow{QS} are opposite rays, as are \overrightarrow{QP} and \overrightarrow{QR} .

Extra Example 3

- Sketch a plane and two intersecting lines that intersect the plane at separate points.



- Sketch a plane and two intersecting lines that do not intersect the plane.



- Sketch a plane and two intersecting lines that lie in the plane.



Key Question to Ask for Example 3

- Can a line intersect a plane in only two points? Explain. **No; a line can intersect a plane in one point if it does not lie in the plane or in an infinite number of points if it does lie in the plane, but never in only two points.**

Avoiding Common Errors

Students may forget to include the line, segment, or ray symbol above letters. Remind them to be sure to include the correct symbol so they can determine whether the named figure is a line, segment, or ray.

EXAMPLE 2 Name segments, rays, and opposite rays

- Give another name for \overrightarrow{GH} .
- Name all rays with endpoint J . Which of these rays are opposite rays?



Solution

- Another name for \overrightarrow{GH} is \overrightarrow{HG} .
- The rays with endpoint J are \overrightarrow{JE} , \overrightarrow{JG} , \overrightarrow{JF} , and \overrightarrow{JH} . The pairs of opposite rays with endpoint J are \overrightarrow{JE} and \overrightarrow{JF} , and \overrightarrow{JG} and \overrightarrow{JH} .

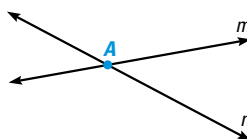


GUIDED PRACTICE for Example 2

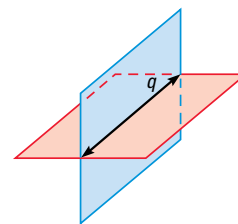
Use the diagram in Example 2.

- Give another name for \overrightarrow{EF} . \overrightarrow{FE}
- Are \overrightarrow{HJ} and \overrightarrow{JH} the same ray? Are \overrightarrow{HJ} and \overrightarrow{HG} the same ray? Explain.
No; the rays have different endpoints; yes; points J and G lie on the same side of H .

INTERSECTIONS Two or more geometric figures *intersect* if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.

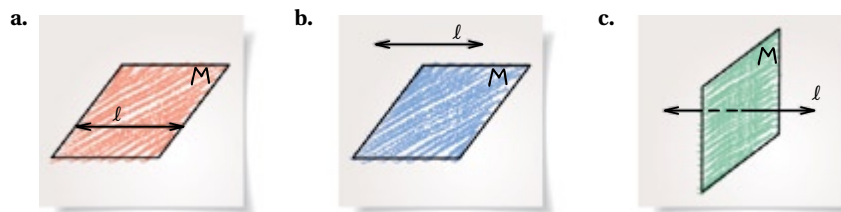


The intersection of two different planes is a line.

EXAMPLE 3 Sketch intersections of lines and planes

- Sketch a plane and a line that is in the plane.
- Sketch a plane and a line that does not intersect the plane.
- Sketch a plane and a line that intersects the plane at a point.

Solution



Differentiated Instruction

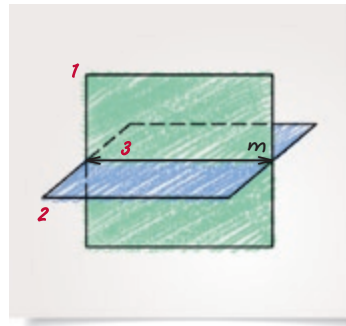
Kinesthetic Learners While discussing **Example 3**, have students simulate the three cases by using a sheet of paper to represent the plane and a pen or pencil to represent the line. Then have students experiment to see that these are the only three possibilities involving a plane and a line. See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 4 Sketch intersections of planes

Sketch two planes that intersect in a line.

Solution

- STEP 1** Draw a vertical plane. Shade the plane.
- STEP 2** Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.
- STEP 3** Draw the line of intersection.

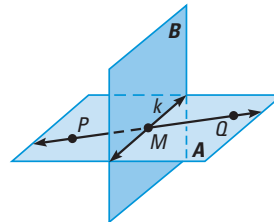


✓ GUIDED PRACTICE for Examples 3 and 4

4. Sketch two different lines that intersect a plane at the same point. **See margin.**

Use the diagram at the right.

5. Name the intersection of \overleftrightarrow{PQ} and line k . **point M**
6. Name the intersection of plane A and plane B. **line k**
7. Name the intersection of line k and plane A. **line k**



1.1 EXERCISES

HOMEWORK KEY

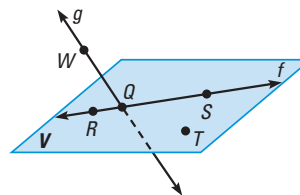
- = See **WORKED-OUT SOLUTIONS**
Exs. 15, 19, and 43
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 7, 13, 16, and 43

SKILL PRACTICE

- A** 1. **VOCABULARY** Write in words what each of the following symbols means.
- a. Q **point Q** b. \overline{MN} **line segment MN** c. \overrightarrow{ST} **ray ST** d. \overleftrightarrow{FG} **line FG**
2. ★ **WRITING** Compare collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? *Explain.* **See margin.**

NAMING POINTS, LINES, AND PLANES In Exercises 3–7, use the diagram.

3. Give two other names for \overleftrightarrow{WQ} . **\overleftrightarrow{QW} , line g**
4. Give another name for plane V. **Sample answer: plane RTS**
5. Name three points that are collinear. Then name a fourth point that is *not* collinear with these three points. **Sample answer: points R, Q, S; point T**
6. Name a point that is *not* coplanar with R, S, and T. **point W**
7. ★ **WRITING** Is point W coplanar with points Q and R? *Explain.*
Yes; through any three points not on the same line, there is exactly one plane.



EXAMPLE 1
for Exs. 3–7

Extra Example 4

Sketch two planes that do not intersect in a line.



Key Question to Ask for Example 4

- Can two planes intersect in a segment? Explain. **No; since the intersecting planes extend without end, their intersection must be a line.**

Closing the Lesson

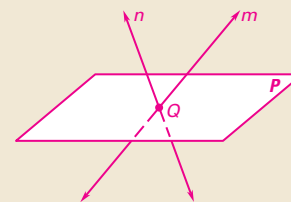
Have students summarize the major points of the lesson and answer the Essential Question: How do you name geometric figures?

- You can sketch and name points, lines, planes, segments, and rays.
- A line may intersect a plane in one point or lie in the plane.
- The intersection of two planes is a line.

Use one letter to name a point, two letters to name the endpoints of a segment, two letters to name the endpoint and one other point of a ray, and two letters to name any two points of a line. Use three letters for three noncollinear points in a plane to name the plane.

Guided Practice

4. **Sample:**



Skill Practice

2. Yes; no; collinear points are points that lie on the same line and therefore in the same plane, while coplanar points lie in the same plane but not necessarily on the same line.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1: SRH p. SR 8 Exs. 1–6
Exs. 1–16, 17–27 odd, 40–44

Average:

Day 1:
Exs. 1, 2, 3–11 odd, 12–16, 20–26,
27–37 odd, 40–45

Advanced:

Day 1:
Exs. 1, 5–7, 10, 11, 13–16,
20–38 even, 39–46*

Block:

Exs. 1, 2, 3–11 odd, 12–16, 20–26,
27–37 odd, 40–45 (with the next
lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 14, 21, 40

Average: 5, 9, 14, 20, 41

Advanced: 6, 13, 22, 26, 42

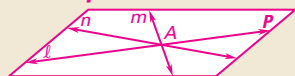
Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

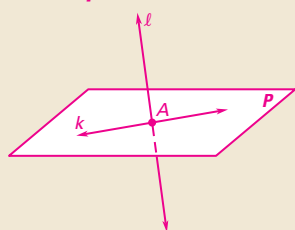
Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

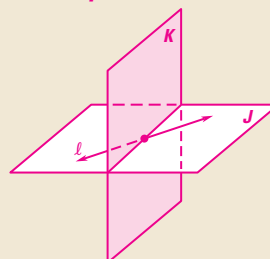
14. Sample:



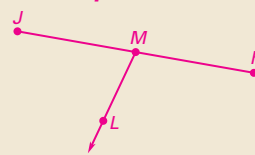
15. Sample:



23. Sample:



25. Sample:



EXAMPLE 2

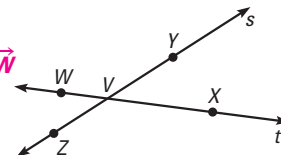
for Exs. 8–13

10. \vec{VX} and \vec{VW} ,
 \vec{VY} and \vec{VZ}

NAMING SEGMENTS AND RAYS

In Exercises 8–12, use the diagram.

- What is another name for \overline{ZY} ? \overline{YZ}
- Name all rays with endpoint V. \vec{VY} , \vec{VX} , \vec{VZ} , \vec{VW}
- Name two pairs of opposite rays.
- Give another name for \overrightarrow{WV} . \overrightarrow{WX}



- ERROR ANALYSIS** A student says that \overrightarrow{VW} and \overrightarrow{VZ} are opposite rays because they have the same endpoint. Describe the error. Point V must lie between points W and Z, which means the three points must be collinear.
- MULTIPLE CHOICE** Which statement about the diagram at the right is true? **B**
 - A, B, and C are collinear.
 - C, D, E, and G are coplanar.
 - B lies on \overrightarrow{GE} .
 - \overrightarrow{EF} and \overrightarrow{ED} are opposite rays.



EXAMPLES 3 and 4

for Exs. 14–23

SKETCHING INTERSECTIONS

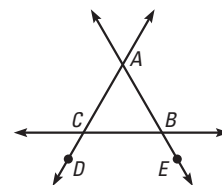
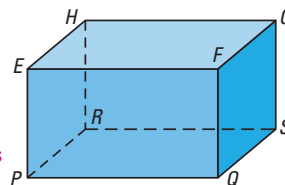
Sketch the figure described. 14, 15. See margin.

- Three lines that lie in a plane and intersect at one point
- One line that lies in a plane, and one line that does not lie in the plane
- MULTIPLE CHOICE** Line AB and line CD intersect at point E, with point E between A and B and between C and D. Which rays are opposite rays? **A**
 - \overrightarrow{EC} and \overrightarrow{ED}
 - \overrightarrow{CE} and \overrightarrow{DE}
 - \overrightarrow{AB} and \overrightarrow{BA}
 - \overrightarrow{AE} and \overrightarrow{BE}

READING DIAGRAMMS

In Exercises 17–22, use the diagram at the right.

- Name the intersection of \overleftrightarrow{PR} and \overleftrightarrow{HR} . point R
- Name the intersection of plane EFG and plane FGS. \vec{FG}
- Name the intersection of plane PQS and plane HGS. \vec{RS}
- Are points P, Q, and F collinear? Are they coplanar? no; yes
- Are points P and G collinear? Are they coplanar? yes; yes
- Name three planes that intersect at point E.
Sample answer: plane PEF, plane PEH, plane HEF
- SKETCHING PLANES** Sketch plane J intersecting plane K. Then draw a line ℓ in plane J that intersects plane K at a single point. See margin.
- NAMING RAYS** Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.
- SKETCHING** Draw three noncollinear points J, K, and L. Sketch \overline{JK} and add a point M on \overline{JK} . Then sketch \overline{ML} . See margin.
- SKETCHING** Draw two points P and Q. Then sketch \overrightarrow{PQ} . Add a point R on the ray so that Q is between P and R. See margin.



6

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

xy ALGEBRA In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. $y = x - 4$; $A(5, 1)$ **on the line** 28. $y = x + 1$; $A(1, 0)$ **not on the line** 29. $y = 3x + 4$; $A(7, 1)$ **not on the line**
 30. $y = 4x + 2$; $A(1, 6)$ **on the line** 31. $y = 3x - 2$; $A(-1, -5)$ **on the line** 32. $y = -2x + 8$; $A(-4, 0)$ **not on the line**

GRAPHING Graph the inequality on a number line. Tell whether the graph is a **segment**, a **ray** or **rays**, a **point**, or a **line**. 33–38. See margin for art.

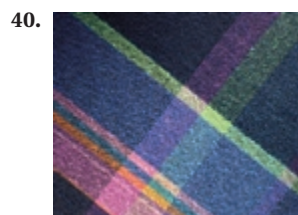
33. $x \leq 3$ **ray** 34. $x \geq -4$ **ray** 35. $-7 \leq x \leq 4$ **segment**
 36. $x \geq 5$ or $x \leq -2$ **rays** 37. $x \geq -1$ or $x \leq 5$ **line** 38. $|x| \leq 0$ **point**

- C** 39. **CHALLENGE** Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch. See margin for art.
- None of the three planes intersect. **possible**
 - The three planes intersect in one line. **possible**
 - The three planes intersect in one point. **possible**
 - Two planes do not intersect. The third plane intersects the other two. **possible**
 - Exactly two planes intersect. The third plane does not intersect the other two. **not possible**

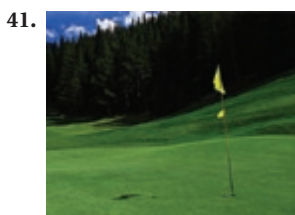
PROBLEM SOLVING

EXAMPLE 3 **A** for Exs. 40–42

EVERYDAY INTERSECTIONS What kind of geometric intersection does the photograph suggest?



intersecting lines



intersection of a line and a plane



intersecting planes

43. Four points are not necessarily coplanar; no; three points determine a unique plane. **B**

- 43. ★ SHORT RESPONSE** Explain why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

- 44. SURVEYING** A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.

- When the tripod is sitting on a level surface, are the tips of the legs coplanar? **yes**
- Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? *Explain.*



26. Sample:



Vocabulary

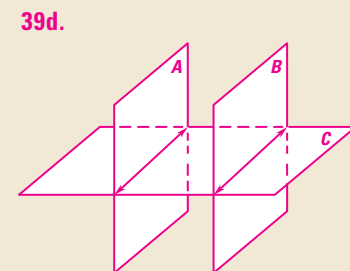
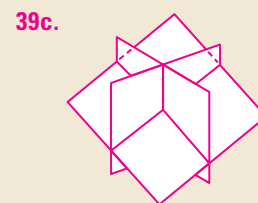
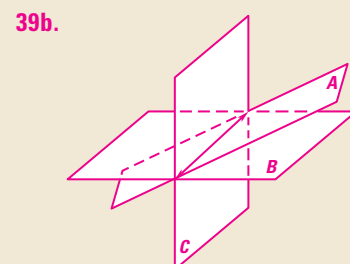
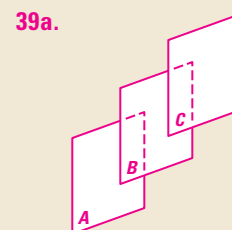
Exercise 13 To help students remember the meaning of the terms **collinear** and **coplanar**, discuss other words that use the prefix *co-*, which means *together*.

Reading Strategy

Exercises 17–22 Point out to students that in the figure different planes are shaded lighter or darker blue to help them visualize the shape.

Teaching Strategy

Exercises 17–22 To help students better understand the diagram, you may wish to have them work in groups and provide each group with a three-dimensional model; a tissue box would work well. Students could label the corner points with a marker.

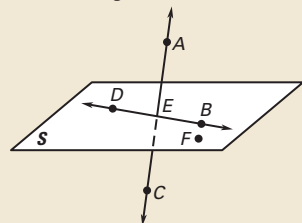


5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Use this figure for Exercises 1–4.



1. Give two other names for \overleftrightarrow{AE} .

\overleftrightarrow{EC} , \overleftrightarrow{AC}

2. Give another name for plane S.

Sample answer: plane DEF

3. Name three collinear points.

Sample answer: A, E, C

4. Name the intersection of \overleftrightarrow{AC} and plane S. E



Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

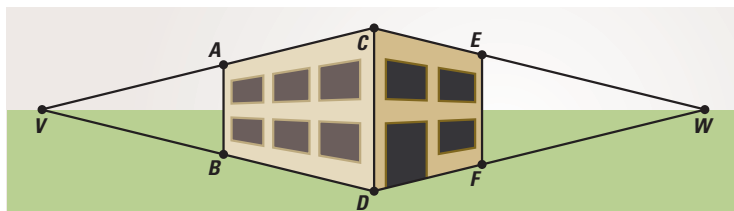
- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

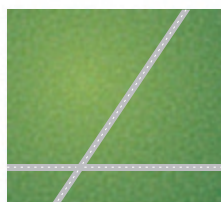
45a–c. See Additional Answers.

45. **MULTI-STEP PROBLEM** In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with two vanishing points. 45a–c. See margin.

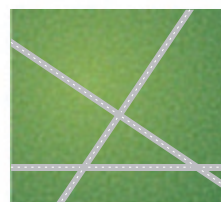


- Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W. Join points E and F to the vanishing point V.
- Label the intersection of \overleftrightarrow{EV} and \overleftrightarrow{AW} as G. Label the intersection of \overleftrightarrow{FV} and \overleftrightarrow{BW} as H.
- Using heavy dashed lines, draw the hidden edges of the house: \overline{AG} , \overline{EG} , \overline{BH} , \overline{FH} , and \overline{GH} .

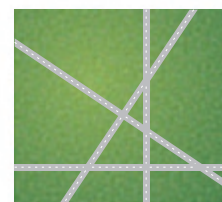
- C** 46. **CHALLENGE** Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.



2 streets



3 streets



4 streets

- A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in the town? 6 streets? **10 traffic lights; 15 traffic lights**
- Describe a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town.
Let n represent the number of streets. To find the additional number of traffic lights needed, find $n - 1$.



1.2 Use Segments and Congruence



Before

You learned about points, lines, and planes.

Now

You will use segment postulates to identify congruent segments.

Why?

So you can calculate flight distances, as in Ex. 33.

Key Vocabulary

- postulate, axiom
- coordinate
- distance
- between
- congruent segments

In Geometry, a rule that is accepted without proof is called a **postulate** or **axiom**. A rule that can be proved is called a **theorem**, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

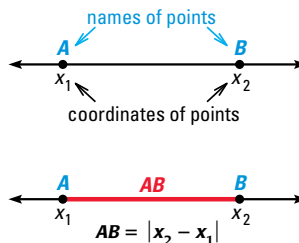
POSTULATE

For Your Notebook

POSTULATE 1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.

The **distance** between points A and B , written as AB , is the absolute value of the difference of the coordinates of A and B .



In the diagrams above, the small numbers in the coordinates x_1 and x_2 are called *subscripts*. The coordinates are read as “x sub one” and “x sub two.”

The distance between points A and B , or AB , is also called the **length** of \overline{AB} .

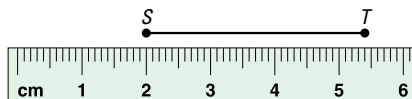
EXAMPLE 1 Apply the Ruler Postulate

Measure the length of \overline{ST} to the nearest tenth of a centimeter.



Solution

Align one mark of a metric ruler with S . Then estimate the coordinate of T . For example, if you align S with 2, T appears to align with 5.4.



$$ST = |5.4 - 2| = 3.4 \quad \text{Use Ruler Postulate.}$$

► The length of \overline{ST} is about 3.4 centimeters.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Solve $3x + 5 + 2x - 4 = 36$. 7
2. Find three cities on this map that appear to be collinear. Chicago, Bloomington, Springfield



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with previous lesson

• See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

What are congruent segments?

Tell students they will learn how to answer this question by finding lengths of segments.

Motivating the Lesson

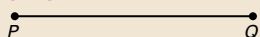
Ask students to think of three places they go that are in a straight line, perhaps their house, a friend's house, and school. Tell them that in this lesson they will learn how to find the third distance between these places when they are given the other two.

3 TEACH

Extra Example 1

Measure the length of \overline{PQ} to the nearest tenth of a centimeter.

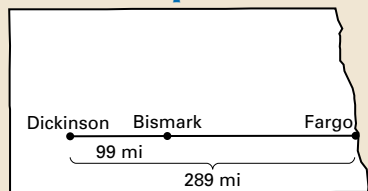
3.2 cm



Key Question to Ask for Example 1

- What if you had aligned S with 1 instead of 2? **The distance is still 3.4, but this time found by using $|4.4 - 1|$.**

Extra Example 2



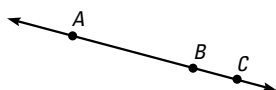
The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Bismarck to Fargo.

190 mi

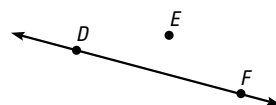
Key Question to Ask for Example 2

- Could you have found the distance if the points were not in a straight line? Explain. **No, because the Segment Addition Postulate would no longer apply.**

ADDING SEGMENT LENGTHS When three points are collinear, you can say that one point is **between** the other two.



Point B is between points A and C.



Point E is not between points D and F.

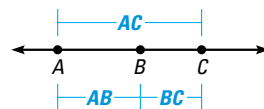
POSTULATE

For Your Notebook

POSTULATE 2 Segment Addition Postulate

If B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



EXAMPLE 2 Apply the Segment Addition Postulate

MAPS The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.



Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

$$LS = LT + TS = 380 + 360 = 740$$

- The distance from Lubbock to St. Louis is about 740 miles.

GUIDED PRACTICE for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

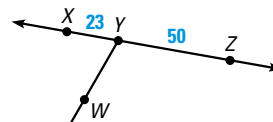
1. \overline{MN} $1\frac{5}{8}$ in.

2. \overline{PQ} $1\frac{3}{8}$ in.

In Exercises 3 and 4, use the diagram shown.

- Use the Segment Addition Postulate to find XZ. **73**

- In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z? Explain your reasoning. **No; Y is not between W and Z.**



Differentiated Instruction

English Learners Have students work problems similar to Example 2 using maps of other countries. In particular, find locations where English learners or their family members have come from.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 3 Find a length

Use the diagram to find GH .



Solution

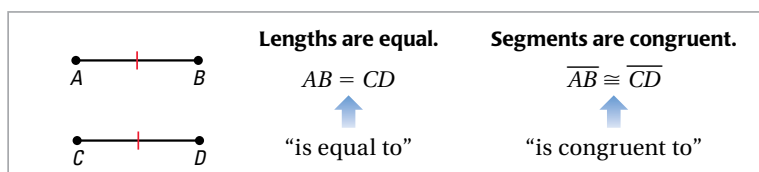
Use the Segment Addition Postulate to write an equation. Then solve the equation to find GH .

$$FH = FG + GH \quad \text{Segment Addition Postulate}$$

$$36 = 21 + GH \quad \text{Substitute 36 for } FH \text{ and 21 for } FG.$$

$$15 = GH \quad \text{Subtract 21 from each side.}$$

CONGRUENT SEGMENTS Line segments that have the same length are called **congruent segments**. In the diagram below, you can say “the length of \overline{AB} is equal to the length of \overline{CD} ,” or you can say “ \overline{AB} is congruent to \overline{CD} .” The symbol \cong means “is congruent to.”



READ DIAGRAMS

In the diagram, the red tick marks indicate that $\overline{AB} \cong \overline{CD}$.

EXAMPLE 4 Compare segments for congruence

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether \overline{JK} and \overline{LM} are congruent.

Solution

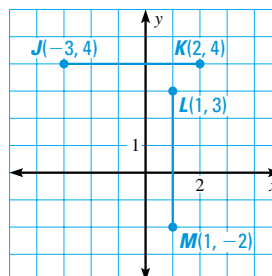
To find the length of a horizontal segment, find the absolute value of the difference of the x -coordinates of the endpoints.

$$JK = |2 - (-3)| = 5 \quad \text{Use Ruler Postulate.}$$

To find the length of a vertical segment, find the absolute value of the difference of the y -coordinates of the endpoints.

$$LM = |-2 - 3| = 5 \quad \text{Use Ruler Postulate.}$$

► \overline{JK} and \overline{LM} have the same length. So, $\overline{JK} \cong \overline{LM}$.



REVIEW USING A COORDINATE PLANE

For help with using a coordinate plane, see p. SR10.



GUIDED PRACTICE for Examples 3 and 4

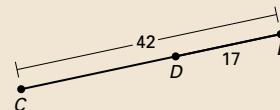
5. Use the diagram at the right to find WX . **107**

6. Plot the points $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether \overline{AB} and \overline{CD} are congruent. **no**



Extra Example 3

Find CD . **25**



Extra Example 4

Graph the points $X(-2, -5)$, $Y(-2, 3)$, $W(-4, 3)$, and $Z(4, 3)$ in a coordinate plane. Are \overline{XY} and \overline{WZ} congruent? **Yes, both equal 8.**

Key Question to Ask for Example 4

- How can you determine which coordinates to subtract? **You subtract the coordinates that are not equal.**

Avoiding Common Errors

Students tend to be lax when using equal and congruent. Stress the concept in the box above Example 4 to point out that lengths, which are numbers, are equal and that segments, which are figures, are congruent.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What are congruent segments?

- To find the distance between two points on a number line, subtract their coordinates, and find the absolute value of the difference.
- The length of a segment is the sum of the lengths of its nonoverlapping parts.
- Congruent segments are segments that have the same length.

Differentiated Instruction

Below Level To stress the difference between equal and congruent, show students two identical one foot rulers. Ask students what the length of each ruler is and have them write the relationship $12 \text{ in.} = 12 \text{ in.}$ (or $1 \text{ ft} = 1 \text{ ft}$). Turn the rulers over, label one as $A-B$ and the other as $C-D$. Tell students to write the relationship between \overline{AB} and \overline{CD} . Students should write $\overline{AB} \cong \overline{CD}$.

See also the *Differentiated Instruction Resources* for more strategies.

1.2 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 13, 17, and 33
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 20, 27, and 34

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1: SRH SR8 Exs. 7–12
Exs. 1–8, 12–23, 32–34

Average:

Day 1:
Exs. 1–5, 7–10, 12–20 even, 21–30, 32–35

Advanced:

Day 1:
Exs. 1–5, 9–11, 15–19 odd, 20–36*

Block:

Exs. 1–5, 7–10, 12–20 even, 21–30, 32–35 (with the previous lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 14, 22, 33

Average: 4, 8, 16, 24, 33

Advanced: 5, 10, 19, 26, 33

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

SKILL PRACTICE

A In Exercises 1 and 2, use the diagram at the right.



1. **VOCABULARY** Explain what \overline{MN} means and what MN means.

\overline{MN} means segment MN while MN is the length of \overline{MN} .

2. **★ WRITING** Explain how you can find PN if you know PQ and QN . How can you find PN if you know MP and MN ?
Find the sum $PQ + QN$; find the difference $MN - MP$.

EXAMPLE 1
for Exs. 3–5

MEASUREMENT Measure the length of the segment to the nearest tenth of a centimeter.

3. 2.1 cm

4. 3.2 cm

5. 3.5 cm

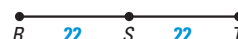
EXAMPLES 2 and 3
for Exs. 6–12

SEGMENT ADDITION POSTULATE Find the indicated length.

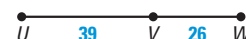
6. Find MP . **23**



7. Find RT . **44**



8. Find UW . **65**



9. Find XY . **23**



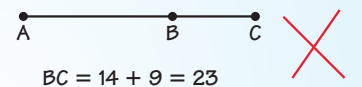
10. Find BC . **15**



11. Find DE . **13**



12. **ERROR ANALYSIS** In the figure at the right, $AC = 14$ and $AB = 9$. Describe and correct the error made in finding BC .
9 should be subtracted from 14, not added; $BC = 14 - 9 = 5$.



EXAMPLE 4
for Exs. 13–19

CONGRUENCE In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

13. $A(0, 1)$, $B(4, 1)$, $C(1, 2)$, $D(1, 6)$; \overline{AB} and \overline{CD} **congruent**

14. $J(-6, -8)$, $K(-6, 2)$, $L(-2, -4)$, $M(-6, -4)$; \overline{JK} and \overline{LM} **not congruent**

15. $R(-200, 300)$, $S(200, 300)$, $T(300, -200)$, $U(300, 100)$; \overline{RS} and \overline{TU} **not congruent**

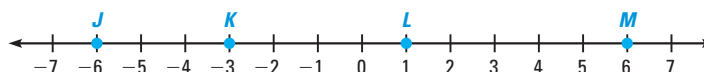
xy ALGEBRA Use the number line to find the indicated distance.

16. JK **3**

17. JL **7**

18. JM **12**

19. KM **9**



- B** 20. **★ SHORT RESPONSE** Use the diagram. Is it possible to use the Segment Addition Postulate to show that $FB > CB$ or that $AC > DB$? Explain.



Yes, since $FB = FC + CB$, then $FB > CB$; no, the relationship between AD and BC is not known.

FINDING LENGTHS In the diagram, points V, W, X, Y, and Z are collinear, $VZ = 52$, $XZ = 20$, and $WX = XY = YZ$. Find the indicated length.

21. WX **10**

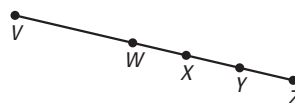
22. VW **22**

23. WY **20**

24. VX **32**

25. WZ **30**

26. VY **42**



27. **★ MULTIPLE CHOICE** Use the diagram.

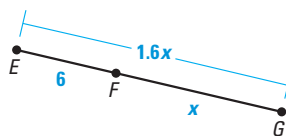
What is the length of \overline{EG} ? **D**

(A) 1

(B) 4.4

(C) 10

(D) 16



xy ALGEBRA Point S is between R and T on \overline{RT} . Use the given information to write an equation in terms of x . Solve the equation. Then find RS and ST.

28. $RS = 2x + 10$

29. $RS = 3x - 16$

30. $RS = 2x - 8$

$ST = x - 4$

$ST = 4x - 8$

$ST = 3x - 10$

$RT = 21$

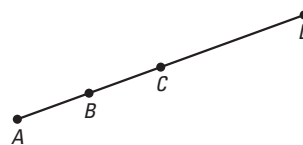
$RT = 60$

$RT = 17$

31. **CHALLENGE** In the diagram, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? *Explain.*

$AC = 6$, $CD = 6$, $AB = 3$, $BC = 3$, $BD = 9$, $AD = 12$; $\frac{2}{3}$;

four of the six segment lengths are greater than 3.

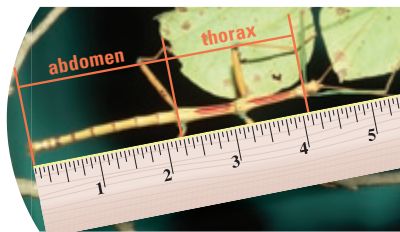


PROBLEM SOLVING

32. $2\frac{1}{4}$ in.,

$1\frac{3}{4}$ in.; $\frac{1}{2}$ in.

A 32. SCIENCE The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. About how much longer is the walkingstick's abdomen than its thorax?



EXAMPLE 2
for Ex. 33

33. MODEL AIRPLANE In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight.



- A** Leave Cape Spear, Newfoundland
- B** Approximate position after about 1 day
- C** Land at Mannin Bay, Ireland, after nearly 38 hours

- a. Find the total distance the model airplane flew. **1883 mi**
- b. The model airplane's flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour. **about 50 mi/h**

Avoiding Common Errors

Exercises 21–26 Students may assume that W is the midpoint of \overline{VZ} . Have them first write all the given measures on the diagram and figure out how long each part is before starting the exercises.

Study Strategy

Exercises 28–30 Have students sketch a diagram and label it with the given lengths. This should help them write an equation.

Internet Reference

Exercise 32 More information about walkingsticks can be found at www.zoo.org/factsheets/walkingstick_viet/walkingstick_viet.html

Animated Geometry

my.hrw.com

An **Animated Geometry** activity is available online for **Exercise 35**. This activity is also part of **Power Presentations**.

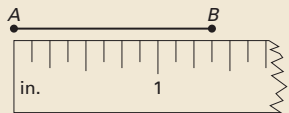
5 ASSESS AND RETEACH

Daily Homework Quiz

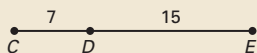
Also available online

1. Measure \overline{AB} to the nearest

$\frac{1}{8}$ inch. **$1\frac{3}{8}$ in.**

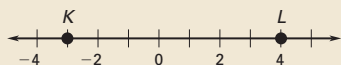


2. Find CE . **22**



3. Plot the points $F(-3, 5)$, $G(2, 5)$, $H(3, 1)$, and $J(3, -3)$ in a coordinate plane. Are \overline{FG} and \overline{HJ} congruent? Explain. **No, $FG = 5$ and $HJ = 4$.**

4. Find KL . **7**



Available at my.hrw.com

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

35a. **Sample:**



36. See Additional Answers.

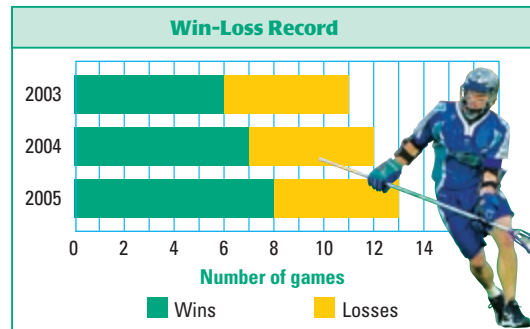
34a. **5, 5, 5; games lost**

34b. **about 45%, about 42%, about 38%**

- B** 34. ★ **SHORT RESPONSE** The bar graph shows the win-loss record for a lacrosse team over a period of three years.

- Use the scale to find the length of the yellow bar for each year. What does the length represent?
- For each year, find the percent of games lost by the team.
- Explain how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.

Sample answer: The sum of the lengths of the bars for wins and losses represents the total number of games played.



35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let A represent the point where the rope is secured at the top of the cliff, let B represent the climber's position, and let C represent the point where the rope is secured at the bottom of the cliff.

- Model** Draw and label a line segment that represents the situation. **See margin.**
- Calculate** If AC is 52 feet and AB is 31 feet, how much farther must the climber descend to reach the bottom of the cliff? **21 ft**

Animated Geometry at my.hrw.com

- C** 36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

See margin.

	City A	City B	City C	City D
City A		?	?	?
City B	?		?	?
City C	?	?		10 mi
City D	?	?	?	

1.3 Use Midpoint and Distance Formulas



Before

You found lengths of segments.

Now

You will find lengths of segments in the coordinate plane.

Why?

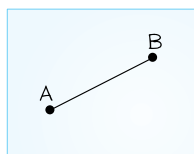
So you can find an unknown length, as in Example 1.

Key Vocabulary

- midpoint
- segment bisector

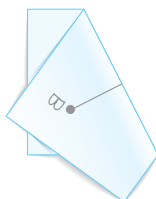
ACTIVITY FOLD A SEGMENT BISECTOR

STEP 1



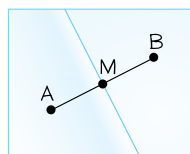
Draw \overline{AB} on a piece of paper.

STEP 2



Fold the paper so that B is on top of A .

STEP 3



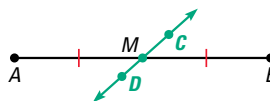
$$AM = MB = \frac{1}{2}AB$$

Label point M . Compare AM , MB , and AB .

MIDPOINTS AND BISECTORS The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.



M is the midpoint of \overline{AB} .
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.



\overleftrightarrow{CD} is a segment bisector of \overline{AB} .
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

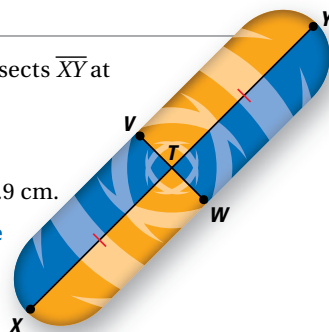
EXAMPLE 1 Find segment lengths

SKATEBOARD In the skateboard design, \overline{VW} bisects \overline{XY} at point T , and $XT = 39.9$ cm. Find XY .

Solution

Point T is the midpoint of \overline{XY} . So, $XT = TY = 39.9$ cm.

$$\begin{aligned} XY &= XT + TY && \text{Segment Addition Postulate} \\ &= 39.9 + 39.9 && \text{Substitute.} \\ &= 79.8 \text{ cm} && \text{Add.} \end{aligned}$$



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

- Find a point between $A(-3, 5)$ and $B(7, 5)$. **Sample: $(2, 5)$**
- Find the average of -11 and 5 . **-3**
- Solve $\frac{x+7}{2} = 5$. **3**
- Find $\sqrt{30}$ to the nearest hundredth. **5.48**
- Find $\sqrt{5} + \sqrt{20}$ to the nearest hundredth. **6.71**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How do you find the distance and the midpoint between two points in the coordinate plane? **Tell students they will learn how to answer this question by using the distance and midpoint formulas.**

Motivating the Lesson

Sue is late for her piano class so she runs diagonally across a rectangular park instead of around it. Tell students that in this lesson they will learn how to find the distance Sue runs and how much shorter the diagonal route is than a route along the outer rim.

3 TEACH

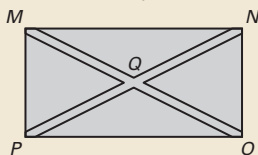
Activity Note

The purpose of this paper folding activity is to have the students locate the midpoint of a segment. They will see that the midpoint is on the fold of the paper. Ask students to measure the distances from the midpoint to each endpoint. Ask what is true of these distances.

They are equal.

Extra Example 1

The figure shows a gate with diagonal braces. \overline{MO} bisects \overline{NP} at Q . If $PQ = 22.6$ in., find PN . **45.2 in.**

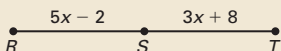


Key Question to Ask for Example 1

- How is XT related to XY ? **It is half of XY .**

Extra Example 2

Point S is the midpoint of \overline{RT} . Find ST . **23**



Key Question to Ask for Example 2

- How do you know that $VM = MW$? **M is a midpoint.**

EXAMPLE 2 Use algebra with segment lengths

xy ALGEBRA Point M is the midpoint of \overline{VW} . Find the length of \overline{VM} .



Solution

STEP 1 Write and solve an equation. Use the fact that $VM = MW$.

$$VM = MW$$

Write equation.

$$4x - 1 = 3x + 3$$

Substitute.

$$x - 1 = 3$$

Subtract $3x$ from each side.

$$x = 4$$

Add 1 to each side.

STEP 2 Evaluate the expression for VM when $x = 4$.

$$VM = 4x - 1 = 4(4) - 1 = 15$$

► So, the length of \overline{VM} is 15.

CHECK Because $VM = MW$, the length of \overline{MW} should be 15. If you evaluate the expression for MW , you should find that $MW = 15$.

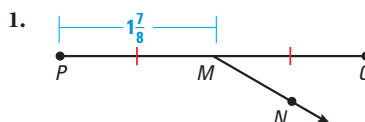
$$MW = 3x + 3 = 3(4) + 3 = 15 \checkmark$$

GUIDED PRACTICE for Examples 1 and 2

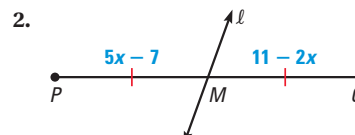
READ DIRECTIONS

Always read direction lines carefully. Notice that this direction line has two parts.

In Exercises 1 and 2, identify the segment bisector of \overline{PQ} . Then find PQ .



$$\overline{MN}; 3\frac{3}{4}$$



$$\text{line } l; 11\frac{5}{7}$$

COORDINATE PLANE You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

KEY CONCEPT

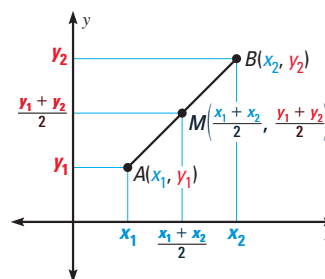
For Your Notebook

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x -coordinates and of the y -coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of \overline{AB} has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



EXAMPLE 3 Use the Midpoint Formula

- a. **FIND MIDPOINT** The endpoints of \overline{RS} are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M .
- b. **FIND ENDPOINT** The midpoint of \overline{JK} is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K .

Solution

- a. **FIND MIDPOINT** Use the Midpoint Formula.

$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

- The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.

- b. **FIND ENDPOINT** Let (x, y) be the coordinates of endpoint K . Use the Midpoint Formula.

STEP 1 Find x .

$$\frac{1+x}{2} = 2$$

$$1+x = 4$$

$$x = 3$$

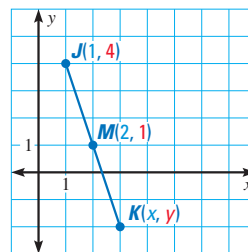
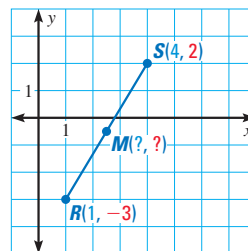
STEP 2 Find y .

$$\frac{4+y}{2} = 1$$

$$4+y = 2$$

$$y = -2$$

- The coordinates of endpoint K are $(3, -2)$.



CLEAR FRACTIONS
Multiply each side of the equation by the denominator to clear the fraction.



GUIDED PRACTICE for Example 3

3. The endpoints of \overline{AB} are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M . **(4, 5)**
4. The midpoint of \overline{VW} is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V . **(-6, -8)**

DISTANCE FORMULA The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

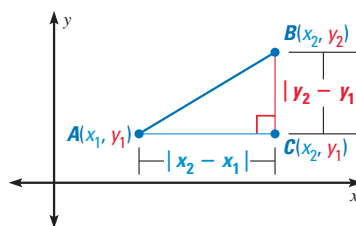
KEY CONCEPT

For Your Notebook

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



READ DIAGRAMS
The red mark at one corner of the triangle shown indicates a right triangle.

Extra Example 3

- a. The endpoints of \overline{GH} are $G(7, -2)$ and $H(-5, -6)$. Find the coordinates of the midpoint P . **(1, -4)**
- b. The midpoint of \overline{AB} is $M(5, 8)$. One endpoint is $A(2, -3)$. Find the coordinates of endpoint B . **(8, 19)**

Key Question to Ask for Example 3

- In part a, which point was used for (x_1, y_1) in the midpoint formula? Could you have used the other point? **$R(1, -3)$; yes, $S(4, 2)$ would work just as well.**

Mathematical Reasoning

In Example 3b, have students note that as you go from $J(1, 4)$ to $M(2, 1)$, the x -coordinate increases by 1 and the y -coordinate decreases by 3. If you follow this pattern to get from M to K , you again get $x = 3$ and $y = -2$.

Differentiated Instruction

Inclusion The subscripts sometimes cause students to make errors in substitution with the Midpoint Formula and the Distance Formula. Remind students that the x -coordinate in the midpoint formula is the average of the two x -values, and the y -coordinate of the midpoint formula is the average of the two y -values. Similarly, the square of the distance in the distance formula is just the square of the difference in the x -values plus the square of the difference in the y -values.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 4

What is the approximate length of \overline{PQ} with endpoints $P(2, 5)$ and $Q(-4, 8)$? **B**

- (A) 3.61 (B) 6.71
(C) 9.0 (D) 13.15

Key Question to Ask for Example 4

- To use the Pythagorean Theorem instead of the distance formula, where would the right angle be located? **(2, -1) or (4, 3)**

Reading Strategy

The symbol \approx is introduced in Example 4. Ask students to differentiate among the symbols $=$, \cong , and \approx .

Closing the Lesson

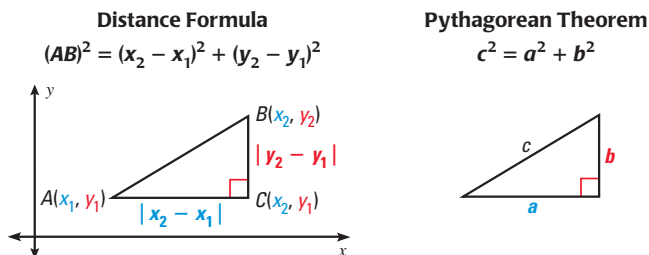
Have students summarize the major points of the lesson and answer the Essential Question: how do you find the distance and the midpoint between two points in the coordinate plane?

- The midpoint of a segment is the point that divides the segment into two congruent parts.
- The length of a segment in the coordinate plane is the distance between its endpoints.

To find the distance between the points (x_1, y_1) and (x_2, y_2) , use the distance formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. To find the midpoint, use the midpoint formula, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

The Distance Formula is based on the *Pythagorean Theorem*, which you will see again when you work with right triangles.



EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES

Drawing a diagram can help you eliminate choices. You can see that choice A is not large enough to be \overline{RS} .

What is the approximate length of \overline{RS} with endpoints $R(2, 3)$ and $S(4, -1)$?

- (A) 1.4 units (B) 4.0 units (C) 4.5 units (D) 6 units

Solution

Use the Distance Formula. You may find it helpful to draw a diagram.

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2} \\ &= \sqrt{(2)^2 + (-4)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

Distance Formula

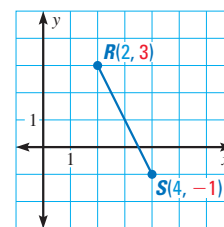
Substitute.

Subtract.

Evaluate powers.

Add.

Use a calculator to approximate the square root.



READ SYMBOLS

The symbol \approx means "is approximately equal to."

► The correct answer is C. (A) (B) (C) (D)



GUIDED PRACTICE for Example 4

5. No. Sample answer: When squaring the differences in the coordinates, you get the same answer as long as you choose the x and y values from the same point.

5. In Example 4, does it matter which ordered pair you choose to substitute for (x_1, y_1) and which ordered pair you choose to substitute for (x_2, y_2) ? **Explain.**

6. What is the approximate length of \overline{AB} , with endpoints $A(-3, 2)$ and $B(1, -4)$? **B**

- (A) 6.1 units (B) 7.2 units (C) 8.5 units (D) 10.0 units

1.3 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 15, 35, and 49

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 23, 34, 41, 42, and 53

SKILL PRACTICE

- 1. VOCABULARY** Copy and complete: To find the length of \overline{AB} , with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the Distance Formula.

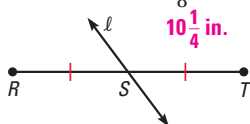
- 2. ★ WRITING** Explain what it means to bisect a segment. Why is it impossible to bisect a line? **Divide a segment into two congruent segments; a line has infinite length.**

EXAMPLE 1

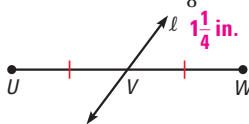
for Exs. 3–10

FINDING LENGTHS Line ℓ bisects the segment. Find the indicated length.

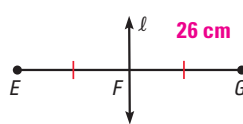
3. Find RT if $RS = 5\frac{1}{8}$ in.



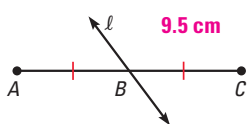
4. Find UV if $VW = \frac{5}{8}$ in.



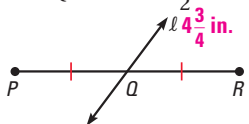
5. Find EG if $EF = 13$ cm.



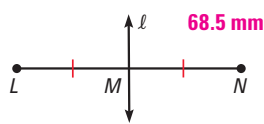
6. Find BC if $AC = 19$ cm.



7. Find QR if $PR = 9\frac{1}{2}$ in.



8. Find LM if $LN = 137$ mm.



- 9. SEGMENT BISECTOR** Line RS bisects \overline{PQ} at point R . Find RQ if $PQ = 4\frac{3}{4}$ inches. **$2\frac{3}{8}$ in.**

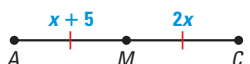
- 10. SEGMENT BISECTOR** Point T bisects \overline{UV} . Find UV if $UT = 2\frac{7}{8}$ inches. **$5\frac{3}{4}$ in.**

EXAMPLE 2

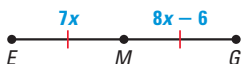
for Exs. 11–16

ALGEBRA In each diagram, M is the midpoint of the segment. Find the indicated length.

11. Find AM . **10**



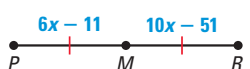
12. Find EM . **42**



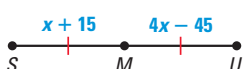
13. Find JM . **1**



14. Find PR . **98**



- 15.** Find SU . **70**



16. Find XZ . **146**



EXAMPLE 3

for Exs. 17–30

FINDING MIDPOINTS Find the coordinates of the midpoint of the segment with the given endpoints.

17. $C(3, 5)$ and $D(7, 5)$ **$(5, 5)$** 18. $E(0, 4)$ and $F(4, 3)$ **$(2, 3\frac{1}{2})$** 19. $G(-4, 4)$ and $H(6, 4)$ **$(1, 4)$**

20. $J(-7, -5)$ and $K(-3, 7)$ **$(-5, 1)$** 21. $P(-8, -7)$ and $Q(11, 5)$ **$(1\frac{1}{2}, -1)$** 22. $S(-3, 3)$ and $T(-8, 6)$ **$(-5\frac{1}{2}, 4\frac{1}{2})$**

- 23. ★ WRITING** Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. Explain your thinking.
 $(\frac{m}{2}, \frac{n}{2})$; when x_2 and y_2 are replaced by zero in the Midpoint Formula and x_1 and y_1 are replaced by m and n the result is $(\frac{m}{2}, \frac{n}{2})$.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1: SRH p. SR2 Exs. 13–20

Exs. 1–16, 48

Day 2:

Exs. 17–27, 31–37, 41, 42, 49–52

Average:

Day 1:

Exs. 1–6, 9, 10, 12–15, 35–40, 48

Day 2:

Exs. 19–24, 28–34, 41–46, 49–53

Advanced:

Day 1:

Exs. 1, 5–10, 14–16, 35–40, 47*, 48

Day 2:

Exs. 20–23, 27–34, 41–46, 49–55*

Block:

Exs. 1–6, 9, 10, 12–15, 19–24, 28–46, 48–53

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 12, 18, 31, 48

Average: 6, 14, 28, 32, 48

Advanced: 8, 16, 30, 33, 48

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

Study Strategy

Exercises 25–30 Point out that in each exercise, the letter M is used for the midpoint of the segment. Discuss how students can check their answers by using the midpoint formula with the coordinates of the point they find and the coordinates of the given endpoint to verify that they get the given coordinates of M .

Avoiding Common Errors

Exercises 31–33 To minimize substitution errors, suggest that students write the distance formula in full before they make any substitutions. Then have them write (x_1, y_1) and (x_2, y_2) below the specific pairs of numbers that they plan to use for the substitutions.



Graphing Calculator

Exercise 41 Suggest that students do paper and pencil calculations to find that $LF = \sqrt{26}$ and $JR = \sqrt{5}$. They can then use the calculator to find the approximate value of $\sqrt{26} - \sqrt{5}$.

24. 8 should be added to 2 and 3 should be added to -1;
 $\left(\frac{8+2}{2}, \frac{3+(-1)}{2}\right)$,
 (5, 1).

EXAMPLE 4
 for Exs. 31–34

42. **Sample answer:** Use the Distance Formula to find PM , then multiply PM by 2 to find PQ ($PM = 5$, $PQ = 10$).

43. $AB = 3\sqrt{5}$,
 $CD = 2\sqrt{10}$;
 not congruent

44. $EF = 5$,
 $GH = \sqrt{41}$;
 not congruent

45. $JK = 8\sqrt{2}$,
 $LM = \sqrt{130}$;
 not congruent **C**

24. **ERROR ANALYSIS** Describe the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.

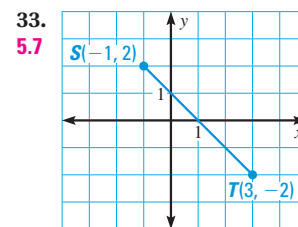
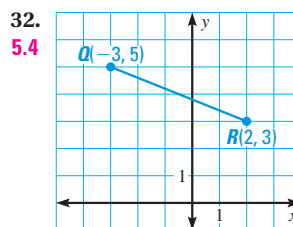
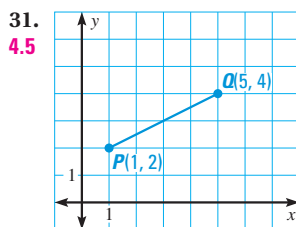
$$\left(\frac{8-2}{2}, \frac{3-(-1)}{2}\right) = (3, 2)$$



FINDING ENDPOINTS Use the given endpoint R and midpoint M of \overline{RS} to find the coordinates of the other endpoint S .

25. $R(3, 0)$, $M(0, 5)$ **(-3, 10)** 26. $R(5, 1)$, $M(1, 4)$ **(-3, 7)** 27. $R(6, -2)$, $M(5, 3)$ **(4, 8)**
 28. $R(-7, 11)$, $M(2, 1)$ **(11, -9)** 29. $R(4, -6)$, $M(-7, 8)$ **(-18, 22)** 30. $R(-4, -6)$, $M(3, -4)$ **(10, -2)**

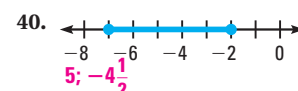
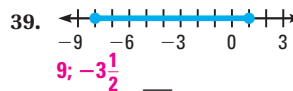
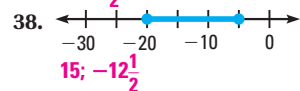
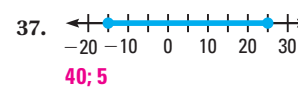
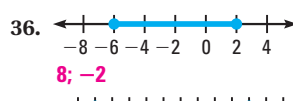
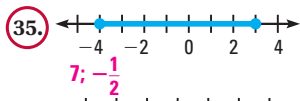
DISTANCE FORMULA Find the length of the segment. Round to the nearest tenth of a unit.



- B** 34. **★ MULTIPLE CHOICE** The endpoints of \overline{MN} are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of \overline{MN} ? **D**

(A) 1.4 units **(B)** 7.2 units **(C)** 13 units **(D)** 18.4 units

NUMBER LINE Find the length of the segment. Then find the coordinate of the midpoint of the segment.



41. **★ MULTIPLE CHOICE** The endpoints of \overline{LF} are $L(-2, 2)$ and $F(3, 1)$. The endpoints of \overline{JR} are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments? **B**

(A) 2.24 **(B)** 2.86 **(C)** 5.10 **(D)** 7.96

42. **★ SHORT RESPONSE** One endpoint of \overline{PQ} is $P(-2, 4)$. The midpoint of \overline{PQ} is $M(1, 0)$. Explain how to find PQ .

COMPARING LENGTHS The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.

43. \overline{AB} : $A(0, 2)$, $B(-3, 8)$ 44. \overline{EF} : $E(1, 4)$, $F(5, 1)$ 45. \overline{JK} : $J(-4, 0)$, $K(4, 8)$
 \overline{CD} : $C(-2, 2)$, $D(0, -4)$ \overline{GH} : $G(-3, 1)$, $H(1, 6)$ \overline{LM} : $L(-4, 2)$, $M(3, -7)$

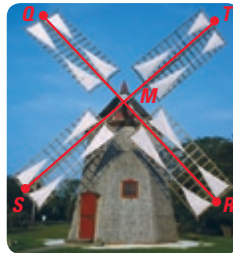
46. **xy ALGEBRA** Points S , T , and P lie on a number line. Their coordinates are 0, 1, and x , respectively. Given $SP = PT$, what is the value of x ? **1.5**

47. **CHALLENGE** M is the midpoint of \overline{JK} , $JM = \frac{x}{8}$, and $JK = \frac{3x}{4} - 6$. Find MK . **1.5**

PROBLEM SOLVING

EXAMPLE 1 [A]
for Ex. 48

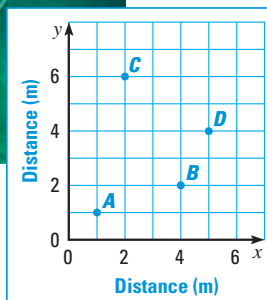
48. **WINDMILL** In the photograph of a windmill, \overline{ST} bisects \overline{QR} at point M . The length of \overline{QM} is $18\frac{1}{2}$ feet. Find \overline{QR} and \overline{MR} . **$\overline{QR} = 37$ ft, $\overline{MR} = 18\frac{1}{2}$ ft**



49. **DISTANCES** A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

See margin for art; 2.85 km

ARCHAEOLOGY The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.



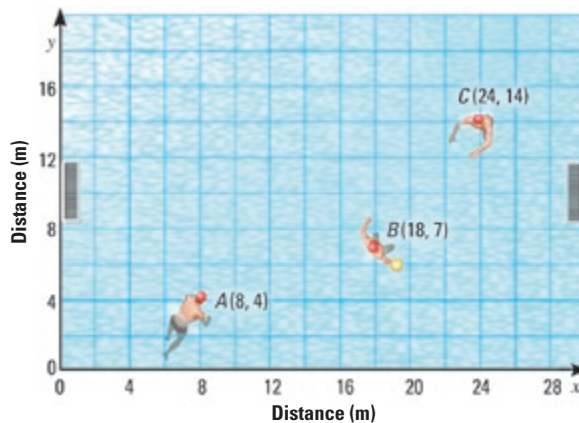
50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.
- a. A and B **3.2 m** b. B and C **4.5 m** c. C and D **3.6 m**
d. A and D **5 m** e. B and D **2.2 m** f. A and C **5.1 m**

51. Which two objects are closest to each other? Which two are farthest apart?

Animated Geometry at my.hrw.com

51. objects
B and D;
objects A and C

- [B] 52. **WATER POLO** The diagram shows the positions of three players during part of a water polo match. Player A throws the ball to Player B, who then throws it to Player C. How far did Player A throw the ball? How far did Player B throw the ball? How far would Player A have thrown the ball if he had thrown it directly to Player C? Round all answers to the nearest tenth of a meter. **10.4 m; 9.2 m; 18.9 m**



Internet Reference

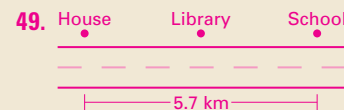
Exercise 50 Additional information about underwater archaeology studies in the United States can be found at www.cr.nps.gov/seac/underh2o.htm

Mathematical Reasoning

Exercise 50 Have students write down the coordinates of each point from the graph before they begin to calculate distances. This will prevent them from having to recalculate the coordinates each time.

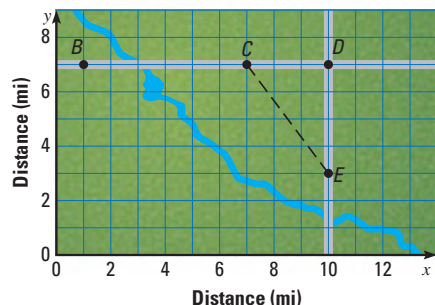
Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Exercise 51**. This activity is also part of **Power Presentations**.



MIXED REVIEW of Problem Solving

1. **MULTI-STEP PROBLEM** The diagram shows existing roads (\overline{BD} and \overline{DE}) and a new road (\overline{CE}) under construction.

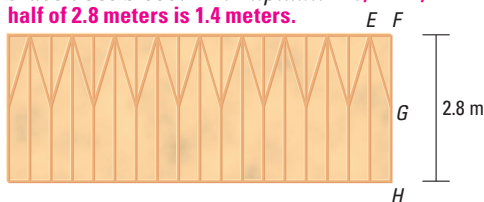


- If you drive from point B to point E on existing roads, how far do you travel? **13 mi**
- If you use the new road as you drive from B to E , about how far do you travel? Round to the nearest tenth of a mile if necessary. **11 mi**
- About how much shorter is the trip from B to E if you use the new road? **2 mi**

2. **GRIDDED ANSWER** Point M is the midpoint of \overline{PQ} . If $PM = 23x + 5$ and $MQ = 25x - 4$, find the length of \overline{PQ} . **217**

3. **GRIDDED ANSWER** You are hiking on a trail that lies along a straight railroad track. The total length of the trail is 5.4 kilometers. You have been hiking for 45 minutes at an average speed of 2.4 kilometers per hour. How much farther (in kilometers) do you need to hike to reach the end of the trail? **3.6 km**

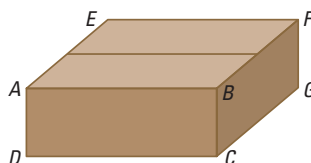
4. **SHORT RESPONSE** The diagram below shows the frame for a wall. \overline{FH} represents a vertical board, and \overline{EG} represents a brace. If $FG = 143$ cm, does the brace bisect \overline{FH} ? If not, how long should \overline{FG} be so that the brace does bisect \overline{FH} ? Explain. **No; 1.4 m; half of 2.8 meters is 1.4 meters.**



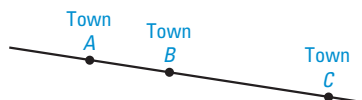
5. **SHORT RESPONSE** Point E is the midpoint of \overline{AB} and the midpoint of \overline{CD} . The endpoints of \overline{AB} are $A(-4, 5)$ and $B(6, -5)$. The coordinates of point C are $(2, 8)$. Find the coordinates of point D . Explain how you got your answer. **See margin.**

6. **OPEN-ENDED** The distance around a figure is its *perimeter*. Choose four points in a coordinate plane that can be connected to form a rectangle with a perimeter of 16 units. Then choose four other points and draw a different rectangle that has a perimeter of 16 units. Show how you determined that each rectangle has a perimeter of 16 units. **See margin.**

7. **SHORT RESPONSE** Use the diagram of a box. What are all the names that can be used to describe the plane that contains points B , F , and C ? Name the intersection of planes ABC and BFE . Explain. **See margin.**



8. **EXTENDED RESPONSE** Jill is a salesperson who needs to visit towns A , B , and C . On the map below, $AB = 18.7$ km and $BC = 2AB$. Assume Jill travels along the road shown.



- Find the distance Jill travels if she starts at Town A , visits Towns B and C , and then returns to Town A . **112.2 km**
- About how much time does Jill spend driving if her average driving speed is 70 kilometers per hour? **about 1.6 h**
- Jill needs to spend 2.5 hours in each town. Can she visit all three towns and return to Town A in an 8 hour workday? Explain. **No; Sample answer: $3(2.5) + 1.6 > 8$**

5. $(0, -8)$; use the coordinates of points A and B and the midpoint formula to find the coordinates of point E , then use the coordinates of points C and E and the midpoint formula to find the coordinates of point D .

6. Check students' work. **Sample answer:** For each rectangle, find the length of each of the four sides and add them.

7. Plane BFC , plane BFG , plane FGC , plane GCB ; \overleftrightarrow{AB} ; \overleftrightarrow{AB} is the edge of the box where plane ABC intersects plane BFE .

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Solve each equation.

1. $5x + 6 + 2x - 14 = 90$ **14**

2. $4x - 30 + x + 25 = 180$ **37**

3. $\frac{x}{3} + 2 = x - 5$ **10.5**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See *Teaching Guide/Lesson Plan*.

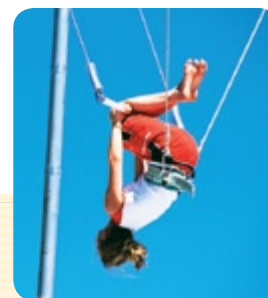
2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you identify whether an angle is acute, right, obtuse, or straight? **Tell students they will learn how to answer this question by learning the definitions of these types of angles and how to find angle measures.**

1.4 Measure and Classify Angles



Before

You named and measured line segments.

Now

You will name, measure, and classify angles.

Why?

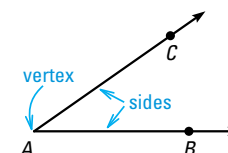
So you can identify congruent angles, as in Example 4.

Key Vocabulary

- **angle**
acute, right, obtuse, straight
- **sides, vertex of an angle**
- **measure of an angle**
- **congruent angles**
- **angle bisector**

An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

The angle with sides \overrightarrow{AB} and \overrightarrow{AC} can be named $\angle BAC$, $\angle CAB$, or $\angle A$. Point A is the vertex of the angle.



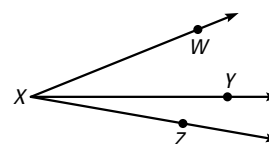
EXAMPLE 1 Name angles

Name the three angles in the diagram.

$\angle WXY$, or $\angle YXW$

$\angle YXZ$, or $\angle ZXY$

$\angle WXZ$, or $\angle ZXW$



You should not name any of these angles $\angle X$ because all three angles have X as their vertex.

MEASURING ANGLES A protractor can be used to approximate the *measure* of an angle. An angle is measured in units called *degrees* ($^\circ$). For instance, the measure of $\angle WXZ$ in Example 1 above is 32° . You can write this statement in two ways.

Words The measure of $\angle WXZ$ is 32° .

Symbols $m\angle WXZ = 32^\circ$

POSTULATE

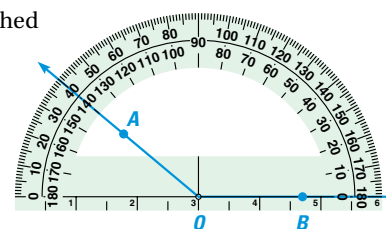
For Your Notebook

POSTULATE 3 Protractor Postulate

Consider \overrightarrow{OB} and a point A on one side of \overrightarrow{OB} .

The rays of the form \overrightarrow{OA} can be matched one to one with the real numbers from 0 to 180.

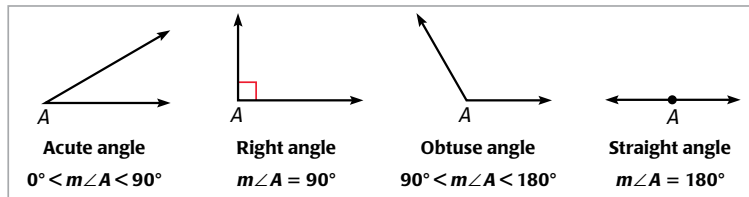
The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for \overrightarrow{OA} and \overrightarrow{OB} .



CLASSIFYING ANGLES Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

READ DIAGRAMS

A red square inside an angle indicates that the angle is a right angle.



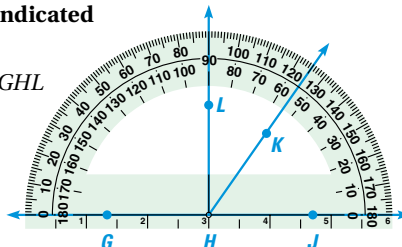
EXAMPLE 2 Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

- a. $\angle KHJ$ b. $\angle GHK$ c. $\angle GHJ$ d. $\angle GHL$

Solution

A protractor has an inner and an outer scale. When you measure an angle, check to see which scale to use.

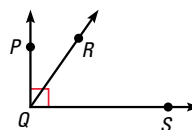


- \overrightarrow{HJ} is lined up with the 0° on the inner scale of the protractor. \overrightarrow{HK} passes through 55° on the inner scale. So, $m\angle KHJ = 55^\circ$. It is an acute angle.
- \overrightarrow{HG} is lined up with the 0° on the outer scale, and \overrightarrow{HK} passes through 125° on the outer scale. So, $m\angle GHK = 125^\circ$. It is an obtuse angle.
- $m\angle GHJ = 180^\circ$. It is a straight angle.
- $m\angle GHL = 90^\circ$. It is a right angle.

Animated Geometry at my.hrw.com

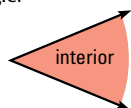
GUIDED PRACTICE for Examples 1 and 2

- Name all the angles in the diagram at the right. Which angle is a right angle?
 $\angle PQR, \angle PQS, \angle RQS, \angle PQS$
- Draw a pair of opposite rays. What type of angle do the rays form? **See margin for art; straight angle.**



READ DIAGRAMS

A point is in the *interior* of an angle if it is between points that lie on each side of the angle.



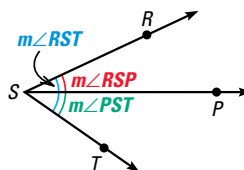
POSTULATE

POSTULATE 4 Angle Addition Postulate

Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$.

For Your Notebook



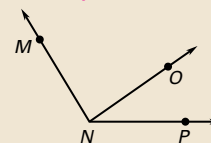
Motivating the Lesson

A piece of glass must be cut to fit the top of a table that has four sides of different lengths. Tell students that in this lesson they will learn how to use angle measures to ensure that the glass will fit perfectly.

3 TEACH

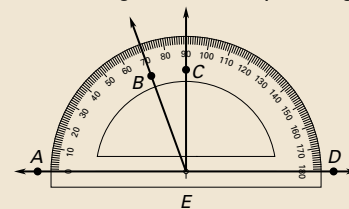
Extra Example 1

Name the three angles in the diagram. **Sample answer:** $\angle MNO$, $\angle ONP$, $\angle MNP$



Extra Example 2

Use the diagram to find the measure of each angle and classify the angle.



- $\angle DEC$ 90° , right
- $\angle DEA$ 180° , straight
- $\angle CEB$ 20° , acute
- $\angle DEB$ 110° , obtuse

Animated Geometry
my.hrw.com

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.



Differentiated Instruction

Inclusion Students may have difficulty remembering which angles are acute and which are obtuse. Have students remember that "acute" means "sharp," and relate this to the fact that the sides of an acute angle form a sharp point.

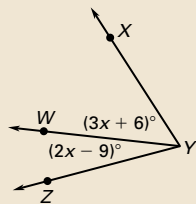
See also the *Differentiated Instruction Resources* for more strategies.

Study Strategy

Have students make up a phrase for remembering AROS, which lists acute, right, obtuse, and straight angles in order of size. One possibility is "All Rain Or Snow."

Extra Example 3

If $m\angle XYZ = 72^\circ$, find $m\angle XYW$ and $m\angle ZYW$. **51°, 21°**



Key Question to Ask for Example 3

- How can you check that the answer of 56° and 89° is correct?

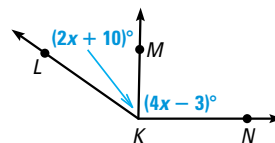
See if the angle measures add up to 145° .

Teaching Strategy

Tell students that we use the \cong symbol to state that a pair of geometric figures have the same size and shape. It makes no sense to write $50^\circ \cong 50^\circ$ or $\frac{6}{2} \cong 3$ since measures and numbers are not geometric figures.

EXAMPLE 3 Find angle measures

xy ALGEBRA Given that $m\angle LKN = 145^\circ$, find $m\angle LKM$ and $m\angle MKN$.



Solution

STEP 1 Write and solve an equation to find the value of x .

$$\begin{aligned} m\angle LKN &= m\angle LKM + m\angle MKN && \text{Angle Addition Postulate} \\ 145^\circ &= (2x + 10)^\circ + (4x - 3)^\circ && \text{Substitute angle measures.} \\ 145 &= 6x + 7 && \text{Combine like terms.} \\ 138 &= 6x && \text{Subtract 7 from each side.} \\ 23 &= x && \text{Divide each side by 6.} \end{aligned}$$

STEP 2 Evaluate the given expressions when $x = 23$.

$$m\angle LKM = (2x + 10)^\circ = (2 \cdot 23 + 10)^\circ = 56^\circ$$

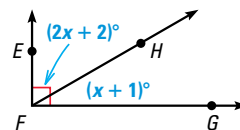
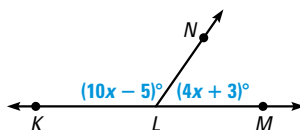
$$m\angle MKN = (4x - 3)^\circ = (4 \cdot 23 - 3)^\circ = 89^\circ$$

► So, $m\angle LKM = 56^\circ$ and $m\angle MKN = 89^\circ$.

GUIDED PRACTICE for Example 3

Find the indicated angle measures.

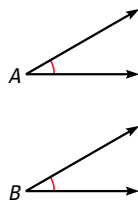
- Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$. **125°, 55°**
- Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$. **60°, 30°**



CONGRUENT ANGLES Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that "the measure of angle A is equal to the measure of angle B," or you can say "angle A is congruent to angle B."

READ DIAGRAMS

Matching arcs are used to show that angles are congruent. If more than one pair of angles are congruent, double arcs are used, as in Example 4.



Angle measures are equal.

$$m\angle A = m\angle B$$



"is equal to"

Angles are congruent.

$$\angle A \cong \angle B$$



"is congruent to"

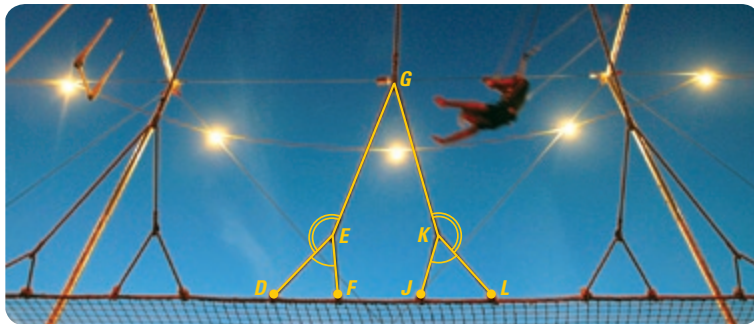
Differentiated Instruction

Advanced Students have learned what it means for segments to be congruent and for angles to be congruent. Ask students to conjecture what it would mean for two triangles to be congruent, and then for two rectangles. Have students extend their conjectures to include any kind of polygon.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 4 Identify congruent angles

TRAPEZE The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If $m\angle DEG = 157^\circ$, what is $m\angle GKL$?



Solution

There are two pairs of congruent angles:

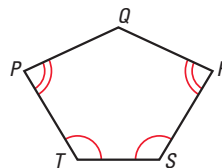
$$\angle DEF \cong \angle JKL \text{ and } \angle DEG \cong \angle GKL.$$

Because $\angle DEG \cong \angle GKL$, $m\angle DEG = m\angle GKL$. So, $m\angle GKL = 157^\circ$.

✓ GUIDED PRACTICE for Example 4

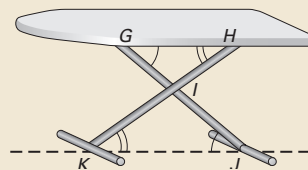
Use the diagram shown at the right.

- Identify all pairs of congruent angles in the diagram. $\angle T$ and $\angle S$, $\angle P$ and $\angle R$
- In the diagram, $m\angle PQR = 130^\circ$, $m\angle QRS = 84^\circ$, and $m\angle TSR = 121^\circ$. Find the other angle measures in the diagram. $m\angle PTS = 121^\circ$, $m\angle QPT = 84^\circ$



Extra Example 4

The figure shows angles formed by the legs of an ironing board. Identify the congruent angles. If $m\angle HGI = 40^\circ$, what is $m\angle GJK$?



$$\angle HGI \cong \angle GJK, \angle GHK \cong \angle HKJ, 40^\circ$$

Key Question to Ask for Example 4

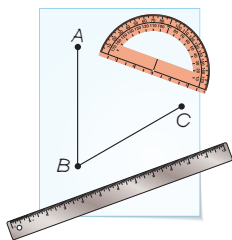
- What is true about the measures of congruent angles? **They are equal.**

Activity Note

The purpose of this folding activity is to show students that an angle bisector divides an angle into two congruent angles. Ask students what the fold represents. **the angle bisector**

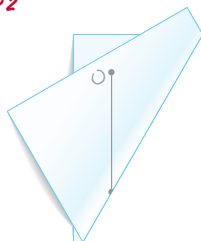
ACTIVITY FOLD AN ANGLE BISECTOR

STEP 1



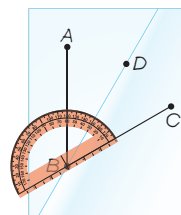
Use a straightedge to draw and label an acute angle, $\angle ABC$.

STEP 2



Fold the paper so that \vec{BC} is on top of \vec{BA} .

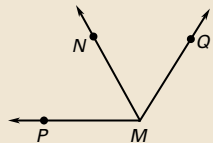
STEP 3



$m\angle ABD = m\angle DBC = \frac{1}{2} m\angle ABC$
Draw a point D on the fold inside $\angle ABC$. Then measure $\angle ABD$, $\angle DBC$, and $\angle ABC$. What do you observe?

Extra Example 5

In the diagram, \overrightarrow{MN} bisects $\angle PMQ$, and $m\angle PMQ = 122^\circ$. Find $m\angle PMN$. **61°**



Key Question to Ask for Example 5

- Is there another method that could be used to find $m\angle XYZ$?

Yes; by the definition of angle

bisector $m\angle XYW = \frac{1}{2}m\angle XYZ$.

Thus $m\angle XYZ = 2m\angle XYW$ or $m\angle XYZ = 2 \cdot 18 = 36^\circ$.

Closing the Lesson

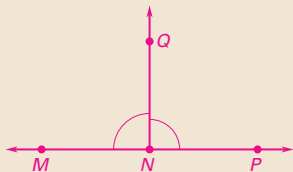
Have students summarize the major points of the lesson and answer the Essential Question: How do you identify whether an angle is acute, right, obtuse, or straight?

- An angle is formed by two rays that have the same endpoint.
- Angles are classified as acute, right, obtuse, or straight.
- An angle bisector divides an angle into two congruent angles.

Acute angles have measure between 0° and 90° , right angles have measure 90° , obtuse angles have measures between 90° and 180° , and straight angles have measure 180° .

Guided Practice

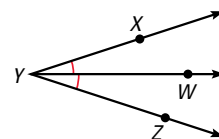
7.



An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on the previous page, \overrightarrow{BD} bisects $\angle ABC$. So, $\angle ABD \cong \angle DBC$ and $m\angle ABD = m\angle DBC$.

EXAMPLE 5 Double an angle measure

In the diagram at the right, \overrightarrow{YW} bisects $\angle XYZ$, and $m\angle XYW = 18^\circ$. Find $m\angle XYZ$.



Solution

By the Angle Addition Postulate, $m\angle XYZ = m\angle XYW + m\angle WYZ$. Because \overrightarrow{YW} bisects $\angle XYZ$, you know that $\angle XYW \cong \angle WYZ$.

So, $m\angle XYW = m\angle WYZ$, and you can write

$$m\angle XYZ = m\angle XYW + m\angle WYZ = 18^\circ + 18^\circ = 36^\circ.$$



GUIDED PRACTICE for Example 5

- Angle MNP is a straight angle, and \overrightarrow{NQ} bisects $\angle MNP$. Draw $\angle MNP$ and \overrightarrow{NQ} . Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles. **See margin for art; 90° .**

1.4 EXERCISES

HOMEWORK KEY

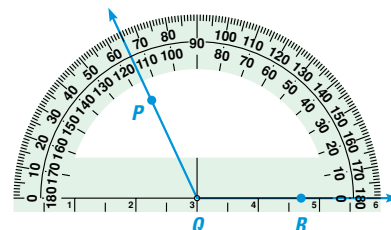
○ = See **WORKED-OUT SOLUTIONS**
Exs. 15, 23, and 53

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 21, 27, 43, and 62

SKILL PRACTICE

A

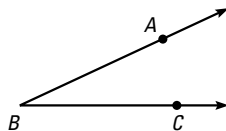
- VOCABULARY** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight. **See margin.**
- ★ WRITING** Explain how to find the measure of $\angle PQR$, shown at the right. **The measure of $\angle PQR$ is equal to the absolute value of the difference between the degree measures for \overrightarrow{QP} and \overrightarrow{QR} .**



EXAMPLE 1
for Exs. 3–6

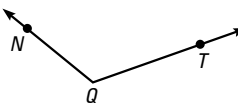
NAMING ANGLES AND ANGLE PARTS In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.

3.



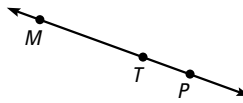
$\angle ABC, \angle B, \angle CBA; B, \overrightarrow{BA}, \overrightarrow{BC}$

4.



$\angle NQT, \angle Q, \angle TQN; Q, \overrightarrow{QN}, \overrightarrow{QT}$

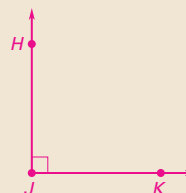
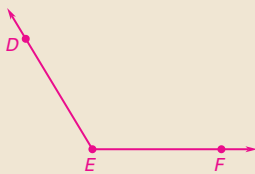
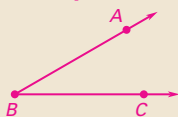
5.



$\angle MTP, \angle T, \angle PTM; T, \overrightarrow{TM}, \overrightarrow{TP}$

Skill Practice

1. Sample:



EXAMPLE 2
for Exs. 7–21

6. **NAMING ANGLES** Name three different angles in the diagram at the right. $\angle QRT$, $\angle QRS$, $\angle SRT$

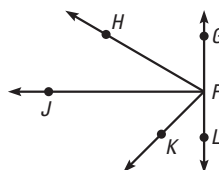


CLASSIFYING ANGLES Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.

7. $m\angle W = 180^\circ$ **straight** 8. $m\angle X = 30^\circ$ **acute** 9. $m\angle Y = 90^\circ$ **right** 10. $m\angle Z = 95^\circ$ **obtuse**

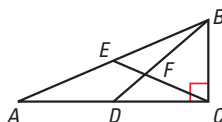
MEASURING ANGLES Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.

11. $\angle JFL$ **90° ; right** 12. $\angle GFH$ **60° ; acute**
13. $\angle GFK$ **135° ; obtuse** 14. $\angle GFL$ **180° ; straight**



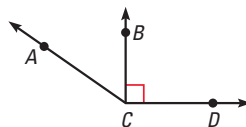
NAMING AND CLASSIFYING Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.

15. $\angle ACB$ 16. $\angle ABC$
17. $\angle BFD$ 18. $\angle AEC$
19. $\angle BDC$ 20. $\angle BEC$



21. **★ MULTIPLE CHOICE** Which is a correct name for the obtuse angle in the diagram? **B**

- (A) $\angle ACB$ (B) $\angle ACD$
(C) $\angle BCD$ (D) $\angle C$



15–20. Sample answers are given.

15. $\angle BCA$; right

16. $\angle CBA$; acute

17. $\angle DFB$; straight

18. $\angle CEA$; obtuse

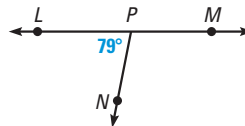
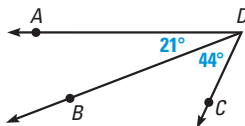
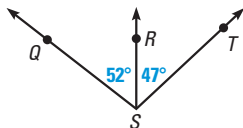
19. $\angle CDB$; acute

20. $\angle CEB$; acute

EXAMPLE 3
for Exs. 22–27

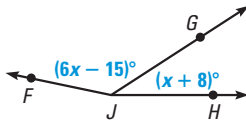
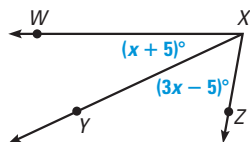
ANGLE ADDITION POSTULATE Find the indicated angle measure.

22. $m\angle QST = ?$ **99°** 23. $m\angle ADC = ?$ **65°** 24. $m\angle NPM = ?$ **101°**



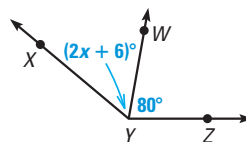
xy ALGEBRA Use the given information to find the indicated angle measure.

25. Given $m\angle WXZ = 80^\circ$, find $m\angle YXZ$. **55°** 26. Given $m\angle FJH = 168^\circ$, find $m\angle FJG$. **135°**



27. **★ MULTIPLE CHOICE** In the diagram, the measure of $\angle XYZ$ is 140° . What is the value of x ? **A**

- (A) 27 (B) 33
(C) 67 (D) 73



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:

Exs. 1–21

Day 2:

Exs. 22–39, 51–58

Average:

Day 1:

Exs. 1, 2, 4–6, 9–18, 21, 44–48

Day 2:

Exs. 22–32, 33–43 odd, 51–62

Advanced:

Day 1:

Exs. 1, 2, 4–6, 8–10, 12–14, 18–21, 44–48

Day 2:

Exs. 24, 26–28, 30–32, 34–42 even, 43, 49–63*

Block:

Exs. 1, 2, 4–6, 9–18, 21–32, 33–43 odd, 44–48, 51–62

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 12, 22, 28, 53

Average: 4, 18, 24, 30, 54

Advanced: 5, 20, 26, 31, 55

Extra Practice

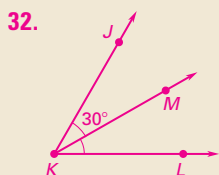
- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

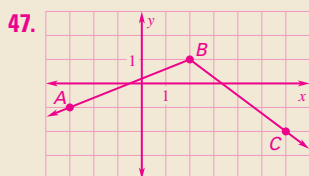
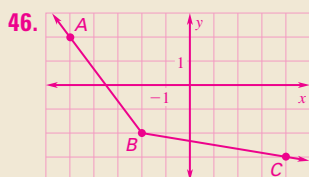
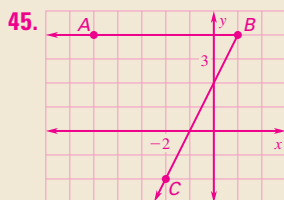
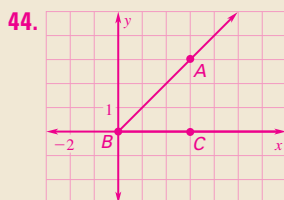
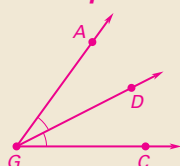
An easily-readable reduced practice page can be found at the beginning of this chapter.

Avoiding Common Errors

Exercise 21 Some students may answer hastily and select choice D. Point out that one-letter notation cannot give the correct answer since all three of the angles have vertex C.



39. Sample:

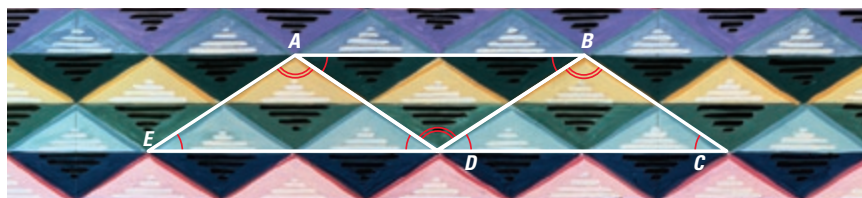


EXAMPLE 4
for Ex. 28

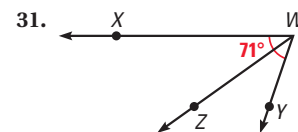
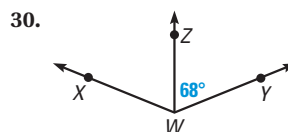
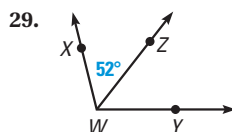
28. $\angle AED \cong$
 $\angle ADE \cong$
 $\angle ABD \cong$
 $\angle DAB \cong$
 $\angle BDC \cong$
 $\angle BCD$, and
 $\angle EAD \cong$
 $\angle ADB \cong$
 $\angle DBC$; 34° , 112°

EXAMPLE 5
for Exs. 29–32

28. **CONGRUENT ANGLES** In the photograph below, $m\angle AED = 34^\circ$ and $m\angle EAD = 112^\circ$. Identify the congruent angles in the diagram. Then find $m\angle BDC$ and $m\angle ADB$.



ANGLE BISECTORS Given that \overrightarrow{WZ} bisects $\angle XWY$, find the two angle measures not given in the diagram.

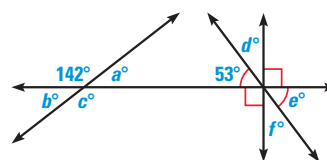


$m\angle XWY = 104^\circ$, $m\angle ZWY = 52^\circ$ $m\angle XWY = 136^\circ$, $m\angle XWZ = 68^\circ$ $m\angle XWZ = 35.5^\circ$, $m\angle YWZ = 35.5^\circ$

32. **ERROR ANALYSIS** \overrightarrow{KM} bisects $\angle JKL$ and $m\angle JKM = 30^\circ$. Describe and correct the error made in stating that $m\angle JKL = 15^\circ$. Draw a sketch to support your answer. **To find $m\angle JKL$, $m\angle JKM$ should be doubled, not halved; $m\angle JKL = 60^\circ$; see margin for art.**

B FINDING ANGLE MEASURES Find the indicated angle measure.

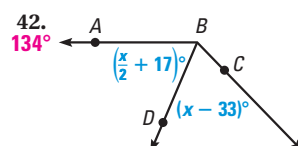
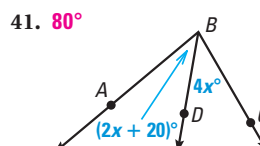
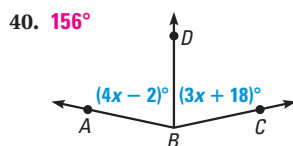
33. a° 38° 34. b° 38°
35. c° 142° 36. d° 37°
37. e° 53° 38. f° 37°



39. **ERROR ANALYSIS** A student states that \overrightarrow{AD} can bisect $\angle AGC$. Describe and correct the student's error. Draw a sketch to support your answer.

If a ray bisects $\angle AGC$, then its endpoint must be point G; see margin for art.

xy ALGEBRA In each diagram, \overrightarrow{BD} bisects $\angle ABC$. Find $m\angle ABC$.



43. **★ SHORT RESPONSE** You are measuring $\angle PQR$ with a protractor. When you line up \overrightarrow{QR} with the 20° mark, \overrightarrow{QP} lines up with the 80° mark. Then you move the protractor so that \overrightarrow{QR} lines up with the 15° mark. What mark does \overrightarrow{QP} line up with? **Explain. 75° ; both angle measures are 5° less.**

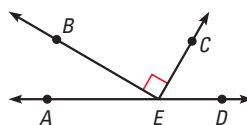
xy ALGEBRA Plot the points in a coordinate plane and draw $\angle ABC$. Classify the angle. Then give the coordinates of a point that lies in the interior of the angle. **44–47. See margin for art.**

44. $A(3, 3)$, $B(0, 0)$, $C(3, 0)$
Acute. Sample answer: $(3, 1)$
46. $A(-5, 2)$, $B(-2, -2)$, $C(4, -3)$
Obtuse. Sample answer: $(0, 0)$

45. $A(-5, 4)$, $B(1, 4)$, $C(-2, -2)$
Acute. Sample answer: $(-2, 0)$
47. $A(-3, -1)$, $B(2, 1)$, $C(6, -2)$
Obtuse. Sample answer: $(2, 0)$

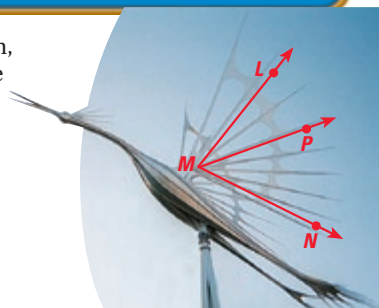
49. **68°.** *Sample answer:* Since $m\angle VSP = 17^\circ$, the $m\angle RSP = 34^\circ$. Since $m\angle RSP = 34^\circ$, $m\angle RSQ = 68^\circ$ which is equal to $m\angle TSQ$, so $m\angle TSQ = 68^\circ$.

48. **xy ALGEBRA** Let $(2x - 12)^\circ$ represent the measure of an acute angle. What are the possible values of x ? **$6 < x < 51$**
49. **CHALLENGE** \overrightarrow{SQ} bisects $\angle RST$, \overrightarrow{SP} bisects $\angle RSQ$, and \overrightarrow{SV} bisects $\angle RSP$. The measure of $\angle VSP$ is 17° . Find $m\angle TSQ$. *Explain.*
50. **FINDING MEASURES** In the diagram, $m\angle AEB = \frac{1}{2} \cdot m\angle CED$, and $\angle AED$ is a straight angle. Find $m\angle AEB$ and $m\angle CED$. **$30^\circ, 60^\circ$**



PROBLEM SOLVING

- A** 51. **SCULPTURE** In the sculpture shown in the photograph, suppose the measure of $\angle LMN$ is 79° and the measure of $\angle PMN$ is 47° . What is the measure of $\angle LMP$? **32°**



52. **MAP** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of 162° . Park Road intersects Sydney Street at an angle of 87° . Find the angle at which Malcom Way intersects Park Road. **75°**



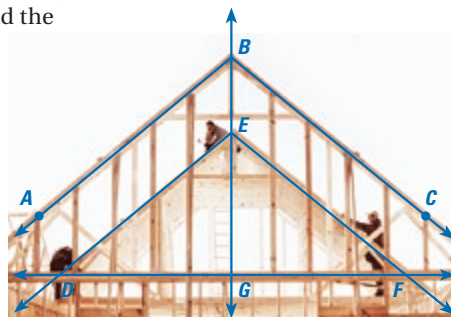
EXAMPLES

4 and 5
for Exs. 53–55

CONSTRUCTION In Exercises 53–55, use the photograph of a roof truss.

53. In the roof truss, \overrightarrow{BG} bisects $\angle ABC$ and $\angle DEF$, $m\angle ABC = 112^\circ$, and $\angle ABC \cong \angle DEF$. Find the measure of the following angles.

- a. $m\angle DEF$ **112°** b. $m\angle ABG$ **56°**
c. $m\angle CBG$ **56°** d. $m\angle DEG$ **56°**



54. In the roof truss, \overrightarrow{GB} bisects $\angle DGF$. Find $m\angle DGE$ and $m\angle FGE$. **$90^\circ, 90^\circ$**
55. Name an example of each of the following types of angles: *acute*, *obtuse*, *right*, and *straight*.

55. **Sample answer:** acute: $\angle ABG$, obtuse: $\angle ABC$, right: $\angle DGE$, straight: $\angle DGF$

Avoiding Common Errors

Exercise 48 Students may write $2x - 12 < 90$ but forget $2x - 12 > 0$. Remind them that they need both inequalities to comply with the definition of an acute angle.

Mathematical Reasoning

Exercise 50 Have students use x to represent $m\angle CED$. Then they can write an equation to find x .

Reading Strategy

Exercise 53 Tell the students that reading and understanding a geometry exercise often requires reading not only the words but also a diagram. The diagram for this exercise contains several segments that are not used in answering the exercise. Students may want to make a separate sketch to show only the parts of the truss that are needed for coming up with the answer.

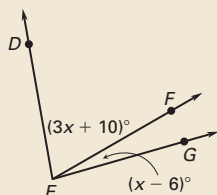
5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Classify each angle as acute, obtuse, right, or straight.

- $m\angle A = 90^\circ$ **right**
- $m\angle B = 62^\circ$ **acute**
- $m\angle C = 119^\circ$ **obtuse**
- If $m\angle DEG = 84^\circ$, find $m\angle FEG$. **14°**



- If \overrightarrow{XY} bisects $\angle ZXW$ and $m\angle ZXY = 36^\circ$, find $m\angle ZXW$. **72°**

Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

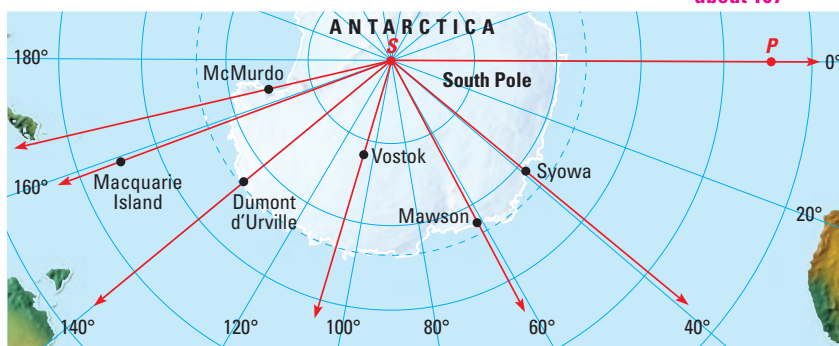
62c. $m\angle DFE = 26^\circ$, $\angle DFE \cong \angle AFB$; $m\angle BFC = 64^\circ$, $m\angle AFB + m\angle BFC = 90^\circ$, $26^\circ + m\angle BFC = 90^\circ$; $\angle AFB$; $m\angle CFD = 64^\circ$, $\angle CFD \cong \angle BFC$; $m\angle AFC = 90^\circ$, it is a right angle ($\angle AFE$ is a straight angle bisected by \overrightarrow{FC}); $m\angle AFD = 154^\circ$, $m\angle AFD = m\angle AFC + m\angle CFD$; $m\angle BFD = 128^\circ$, $m\angle BFD = m\angle BFC + m\angle CFD$.

- B GEOGRAPHY** For the given location on the map, estimate the measure of $\angle PSL$, where P is on the Prime Meridian (0° longitude), S is the South Pole, and L is the location of the indicated research station.

- Macquarie Island **about 158°**
- Mawson **about 62°**

- Dumont d'Urville **about 140°**
- Syowa **about 39°**

- McMurdo **about 167°**
- Vostok **about 107°**

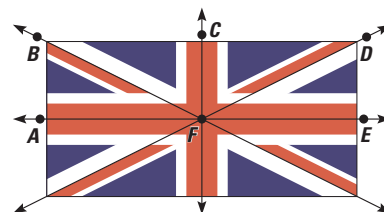


62a. $\angle AFB$,
 $\angle BFC$, $\angle CFD$,
 $\angle DFE$;
 $\angle AFD$, $\angle BFE$;
 $\angle AFC$, $\angle CFE$

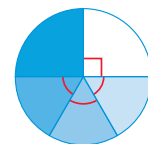
62b.
 $\angle AFB \cong \angle EFD$,
 $\angle AFC \cong \angle EFC$,
 $\angle BFC \cong \angle DFC$

62. **★ EXTENDED RESPONSE** In the flag shown, $\angle AFE$ is a straight angle and \overrightarrow{FC} bisects $\angle AFE$ and $\angle BFD$.

- Which angles are acute? obtuse? right?
- Identify the congruent angles.
- If $m\angle AFB = 26^\circ$, find $m\angle DFE$, $m\angle BFC$, $m\angle CFD$, $m\angle AFC$, $m\angle AFD$, and $m\angle BFD$. Explain. **See margin.**



- C 63. CHALLENGE** Create a set of data that could be represented by the circle graph at the right. Explain your reasoning.
Sample answer: In your pocket you have 4 pennies, 4 nickels, 4 dimes, 6 quarters, 6 dollar bills; nickels, dimes, pennies each represent $\frac{1}{6}$, and quarters and dollar bills each represent $\frac{1}{4}$.



Copy and Bisect Segments and Angles

MATERIALS • compass • straightedge

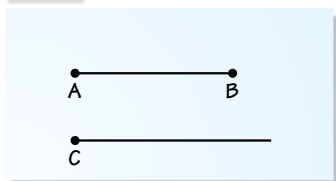
QUESTION How can you copy and bisect segments and angles?

A **construction** is a geometric drawing that uses a limited set of tools, usually a *compass* and *straightedge*. You can use a compass and straightedge (a ruler without marks) to construct a segment that is congruent to a given segment, and an angle that is congruent to a given angle.

EXPLORE 1 Copy a segment

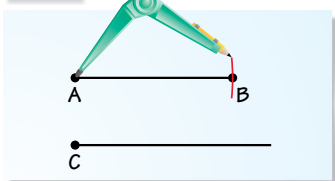
Use the following steps to construct a segment that is congruent to \overline{AB} .

STEP 1



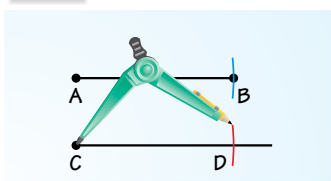
Draw a segment Use a straightedge to draw a segment longer than \overline{AB} . Label point C on the new segment.

STEP 2



Measure length Set your compass at the length of \overline{AB} .

STEP 3

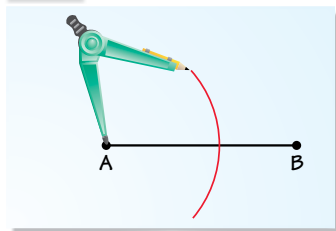


Copy length Place the compass at C . Mark point D on the new segment. $\overline{CD} \cong \overline{AB}$.

EXPLORE 2 Bisect a segment

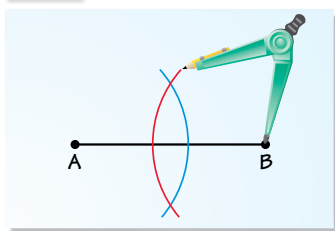
Use the following steps to construct a bisector of \overline{AB} and to find the midpoint M of \overline{AB} . This construction is justified in the lesson *Use Perpendicular Bisectors*, Exercise 27.

STEP 1



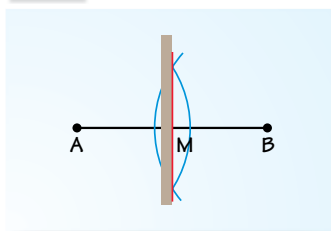
Draw an arc Place the compass at A . Use a compass setting that is greater than half the length of \overline{AB} . Draw an arc.

STEP 2



Draw a second arc Keep the same compass setting. Place the compass at B . Draw an arc. It should intersect the other arc at two points.

STEP 3



Bisect segment Draw a segment through the two points of intersection. This segment bisects \overline{AB} at M , the midpoint of \overline{AB} .

1 PLAN AND PREPARE

Explore the Concept

- Students will copy and bisect a segment and an angle.
- This activity supplements the study of congruent segments and angles, and segment and angle bisectors.

Materials

Each student will need:

- compass
- straightedge

Recommended Time

Work activity: 15 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

In Explore 2, point out to students that it is important not to change the compass setting after they draw the first arc.

In Explore 4, students should note that the figure for Step 2 used a different compass setting from that used in Step 1. The same compass setting for all three arcs will also work.

Alternative Strategy

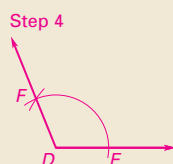
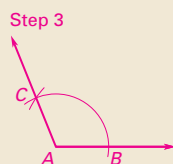
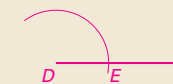
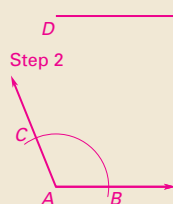
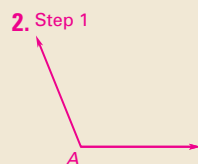
Demonstrate these constructions on the chalkboard or with a figure on the overhead projector.

Key Discovery

There are infinitely many bisectors of a segment but the one that is constructed in this activity is perpendicular to the segment. There is only one bisector of an angle.

3 ASSESS AND RETEACH

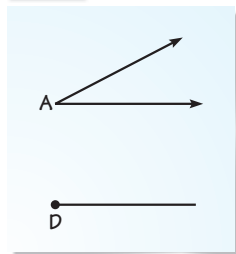
- How many compass settings would you need to construct a segment that is three times as long as a given segment? **one**
- When you construct an angle bisector, are there any restrictions on the radius of the two small arcs you draw in Step 2? Explain. **Yes, the radius should be noticeably greater than half the length of \overline{CB} . Otherwise, you cannot see clearly where the arcs intersect. The arcs will not even intersect if the radius is less than half of \overline{CB} .**



EXPLORE 3 Copy an angle

Use the following steps to construct an angle that is congruent to $\angle A$. In this construction, the *radius* of an arc is the distance from the point where the compass point rests (the *center* of the arc) to a point on the arc drawn by the compass. This construction is justified in the lesson *Use Congruent Triangles*, Example 4.

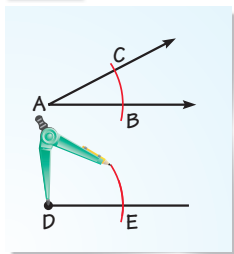
STEP 1



Draw a segment

Draw a segment. Label a point D on the segment.

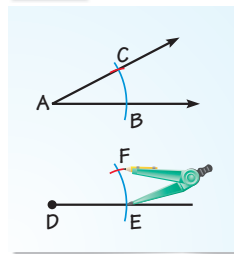
STEP 2



Draw arcs

Draw an arc with center A . Using the same radius, draw an arc with center D .

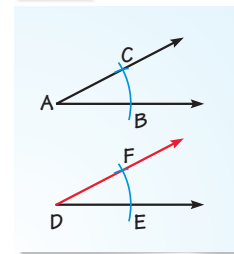
STEP 3



Draw arcs

Label B , C , and E . Draw an arc with radius BC and center E . Label the intersection F .

STEP 4



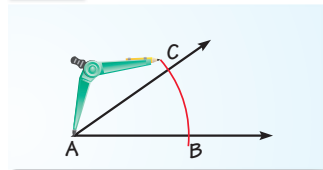
Draw a ray

Draw \overrightarrow{DF} . $\angle EDF \cong \angle BAC$.

EXPLORE 4 Bisect an angle

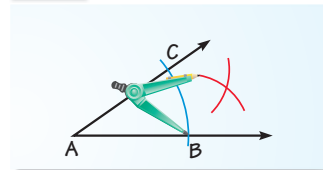
Use the following steps to construct an angle bisector of $\angle A$. This construction is justified in the lesson *Use Congruent Triangles*, Exercise 32.

STEP 1



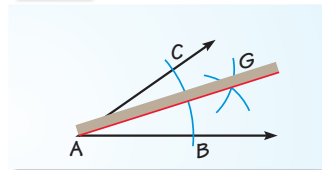
Draw an arc Place the compass at A . Draw an arc that intersects both sides of the angle. Label the intersections C and B .

STEP 2



Draw arcs Place the compass at C . Draw an arc. Then place the compass point at B . Using the same radius, draw another arc.

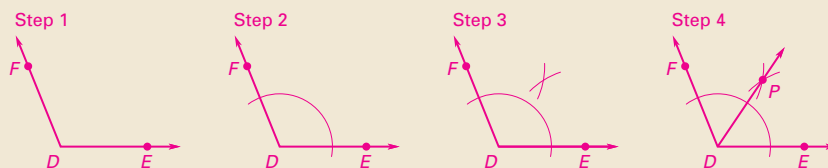
STEP 3



Draw a ray Label the intersection G . Use a straightedge to draw a ray through A and G . \overrightarrow{AG} bisects $\angle A$.

DRAW CONCLUSIONS Use your observations to complete these exercises

- Describe how you could use a compass and a straightedge to draw a segment that is twice as long as a given segment. **Sample answer: Copy the shorter line segment twice end-to-end.**
- Draw an obtuse angle. Copy the angle using a compass and a straightedge. Then bisect the angle using a compass and straightedge. **See margin.**



1.5 Describe Angle Pair Relationships



Before

You used angle postulates to measure and classify angles.

Now

You will use special angle relationships to find angle measures.

Why?

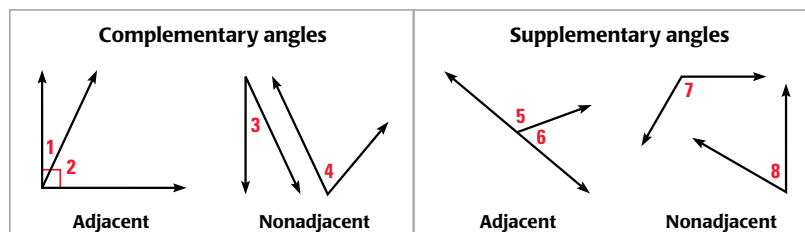
So you can find measures in a building, as in Ex. 53.

Key Vocabulary

- complementary angles
- supplementary angles
- adjacent angles
- linear pair
- vertical angles

Two angles are **complementary angles** if the sum of their measures is 90° . Each angle is the *complement* of the other. Two angles are **supplementary angles** if the sum of their measures is 180° . Each angle is the *supplement* of the other.

Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.

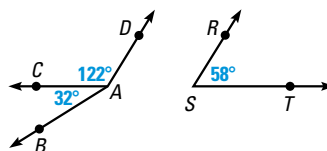


EXAMPLE 1 Identify complements and supplements

AVOID ERRORS

In Example 1, $\angle DAC$ and $\angle DAB$ share a common vertex. But they share common interior points, so they are *not* adjacent angles.

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because $32^\circ + 58^\circ = 90^\circ$, $\angle BAC$ and $\angle RST$ are complementary angles.

Because $122^\circ + 58^\circ = 180^\circ$, $\angle CAD$ and $\angle RST$ are supplementary angles.

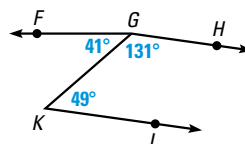
Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent.



GUIDED PRACTICE for Example 1

2. No, they do not share a common vertex; no, they have common interior points.

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles. $\angle FGK$ and $\angle GKL$, $\angle HGL$ and $\angle GKL$, $\angle FGK$ and $\angle HGL$
2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.



1 PLAN AND PREPARE

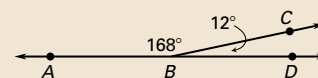
Warm-Up Exercises

Also available online

1. The sum of two numbers is 90 and one number is 4 times the other. Write an equation and solve to find the numbers.

$$x + 4x = 90; 18, 72$$

2. Find $m\angle ABD$. What kind of angle is it? 180° , straight



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with next lesson

• See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How do you identify complementary and supplementary angles?

Tell students they will learn how to answer this question by finding the sum of the measures of two given angles.

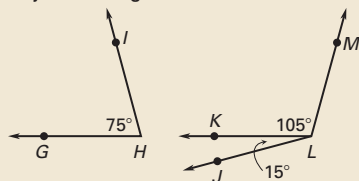
Motivating the Lesson

Parts for furniture that you assemble yourself often include parts that must fit together to form right angles or straight edges. Tell students that in this lesson they will learn terms that are useful in understanding how the angles for such parts are related.

3 TEACH

Extra Example 1

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



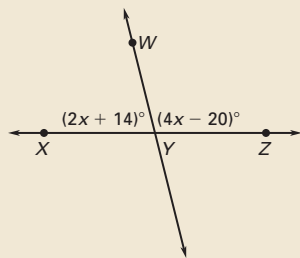
$\angle GHI, \angle JLK; \angle GHI, \angle KLM; \angle JLK, \angle KLM$

Extra Example 2

- Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 17^\circ$, find $m\angle 2$. **73°**
- Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 119^\circ$, find $m\angle 4$. **61°**

Extra Example 3

Two roads intersect to form supplementary angles, $\angle XYW$ and $\angle WYZ$. Find $m\angle XYW$ and $m\angle WYZ$. **$76^\circ, 104^\circ$**



READ DIAGRAMS

Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

EXAMPLE 2 Find measures of a complement and a supplement

- Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68^\circ$, find $m\angle 2$.
- Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56^\circ$, find $m\angle 3$.

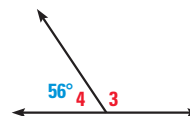
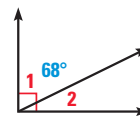
Solution

- You can draw a diagram with complementary adjacent angles to illustrate the relationship.

$$m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 68^\circ = 22^\circ$$

- You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

$$m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 56^\circ = 124^\circ$$

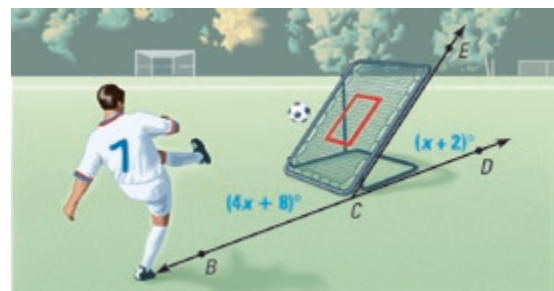


EXAMPLE 3 Find angle measures

READ DIAGRAMS

In a diagram, you can assume that a line that looks straight is straight. In Example 3, B, C, and D lie on \overleftrightarrow{BD} . So, $\angle BCD$ is a straight angle.

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



Solution

STEP 1 Use the fact that the sum of the measures of supplementary angles is 180° .

$$m\angle BCE + m\angle ECD = 180^\circ \quad \text{Write equation.}$$

$$(4x + 8)^\circ + (x + 2)^\circ = 180^\circ \quad \text{Substitute.}$$

$$5x + 10 = 180 \quad \text{Combine like terms.}$$

$$5x = 170 \quad \text{Subtract 10 from each side.}$$

$$x = 34 \quad \text{Divide each side by 5.}$$

STEP 2 Evaluate the original expressions when $x = 34$.

$$m\angle BCE = (4x + 8)^\circ = (4 \cdot 34 + 8)^\circ = 144^\circ$$

$$m\angle ECD = (x + 2)^\circ = (34 + 2)^\circ = 36^\circ$$

► The angle measures are 144° and 36° .



GUIDED PRACTICE for Examples 2 and 3

- Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 8^\circ$, find $m\angle 1$. **82°**
- Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117^\circ$, find $m\angle 4$. **63°**
- $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$. **$26^\circ, 64^\circ$**

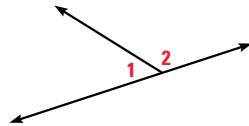
Differentiated Instruction

Inclusion Students may confuse the definitions of *complementary angles* and *supplementary angles*. One memory device is to notice that both of the sets {complementary, supplementary} and $\{90^\circ, 180^\circ\}$ are in alphabetical order and ascending order, respectively.

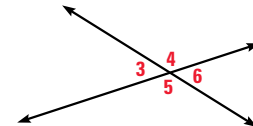
See also the *Differentiated Instruction Resources* for more strategies.

ANGLE PAIRS Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays.



$\angle 1$ and $\angle 2$ are a linear pair.



$\angle 3$ and $\angle 6$ are vertical angles.

$\angle 4$ and $\angle 5$ are vertical angles.

EXAMPLE 4 Identify angle pairs

AVOID ERRORS

In the diagram, one side of $\angle 1$ and one side of $\angle 3$ are opposite rays. But the angles are not a linear pair because they are not adjacent.

Identify all of the linear pairs and all of the vertical angles in the figure at the right.

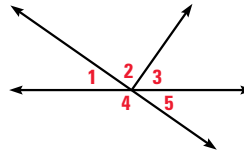
Solution

To find vertical angles, look for angles formed by intersecting lines.

▶ $\angle 1$ and $\angle 5$ are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ $\angle 1$ and $\angle 4$ are a linear pair. $\angle 4$ and $\angle 5$ are also a linear pair.



EXAMPLE 5 Find angle measures in a linear pair

xy ALGEBRA Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.

Solution

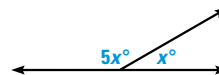
Let x° be the measure of one angle. The measure of the other angle is $5x^\circ$. Then use the fact that the angles of a linear pair are supplementary to write an equation.

$$x^\circ + 5x^\circ = 180^\circ \quad \text{Write an equation.}$$

$$6x = 180 \quad \text{Combine like terms.}$$

$$x = 30 \quad \text{Divide each side by 6.}$$

▶ The measures of the angles are 30° and $5(30^\circ) = 150^\circ$.



DRAW DIAGRAMS

You may find it useful to draw a diagram to represent a word problem like the one in Example 5.

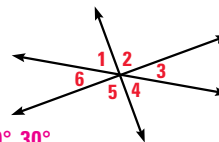
6. No, no adjacent angles have their noncommon sides as opposite rays; $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 6$, these pairs of angles have sides that form two pairs of opposite rays.



GUIDED PRACTICE for Examples 4 and 5

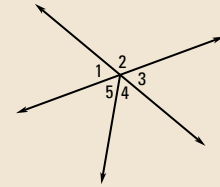
6. Do any of the numbered angles in the diagram at the right form a linear pair? Which angles are vertical angles? *Explain.*

7. The measure of an angle is twice the measure of its complement. Find the measure of each angle. **$60^\circ, 30^\circ$**



Extra Example 4

Identify all of the linear pairs and all of the vertical angles in the figure.



$\angle 2$ and $\angle 3$, $\angle 1$ and $\angle 2$; $\angle 1$ and $\angle 3$

Key Question to Ask for Example 4

• The noncommon sides of $\angle 2$ and $\angle 3$ form an angle. What kind of angle pair will this angle form with $\angle 4$? **vertical**

Extra Example 5

Two angles form a linear pair. The measure of one angle is 3 times the measure of the other angle. Find the measure of each angle. **$45^\circ, 135^\circ$**

Key Question to Ask for Example 5

• How would your equation change if one angle was 5° more than the other? **It would be $x^\circ + (x + 5)^\circ = 180^\circ$.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you identify complementary and supplementary angles?

• **Complementary angles have a sum of 90° .**

• **Supplementary angles have a sum of 180° .**

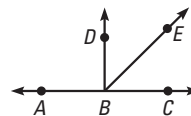
• **Adjacent angles share a vertex but no common interior points, while a linear pair are adjacent angles whose noncommon sides are opposite rays.**

• **Vertical angles are angles whose sides form two pairs of opposite rays.**

Add the measures of the angles. If the sum is 90° , the angles are complementary. If the sum is 180° , the angles are supplementary.

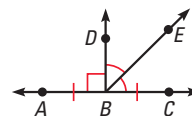
Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you **can conclude** from the diagram at the right:



- All points shown are coplanar.
- Points A, B, and C are collinear, and B is between A and C.
- \overrightarrow{AC} , \overrightarrow{BD} , and \overrightarrow{BE} intersect at point B.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point E lies in the interior of $\angle DBC$.

In the diagram above, you **cannot conclude** that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



1.5 EXERCISES

HOMEWORK KEY

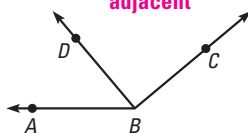
- = See **WORKED-OUT SOLUTIONS**
Exs. 9, 21, and 47
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 16, 30, and 53
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 55

SKILL PRACTICE

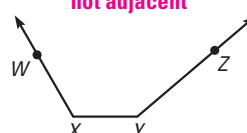
- A** 1. **VOCABULARY** Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? *Explain.* **See margin.**
2. **★ WRITING** Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? *Explain.* **See margin.**

IDENTIFYING ANGLES Tell whether the indicated angles are adjacent.

3. $\angle ABD$ and $\angle DBC$
adjacent



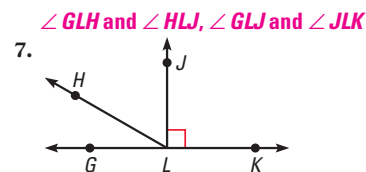
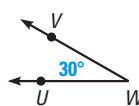
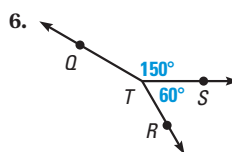
4. $\angle WXY$ and $\angle XYZ$
not adjacent



5. $\angle LQM$ and $\angle NQM$
adjacent



IDENTIFYING ANGLES Name a pair of complementary angles and a pair of supplementary angles.



6. $\angle RTS$ and $\angle UTV$, $\angle QTS$ and $\angle UTV$

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:

Exs. 1–7, 9–15 odd, 16, 17–27 odd, 28–35, 46–54

Average:

Day 1:

Exs. 1, 2, 4–7, 8–28 even, 29, 30–44 even, 46–55

Advanced:

Day 1:

Exs. 1, 2, 5, 7, 11, 15, 16, 19, 25–30, 33–45* odd, 48–56*

Block:

Exs. 1, 2, 4–7, 8–28 even, 29, 30–44 even, 46–55 (with the next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 9, 17, 23, 46

Average: 6, 12, 18, 24, 50

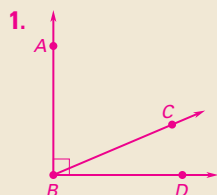
Advanced: 7, 15, 19, 28, 53

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.



1. No. **Sample answer:** Any two angles whose angle measures add up to 90° are complementary, but they do not have to have a common vertex and side.

2. Yes; no. **Sample answer:** To be a linear pair, the noncommon sides of two adjacent angles must be opposite rays, which make a straight angle; supplementary angles need not be linear pairs because they can be nonadjacent.

EXAMPLE 2
for Exs. 8–16

COMPLEMENTARY ANGLES $\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

8. $m\angle 1 = 43^\circ$ **47°** 9. $m\angle 1 = 21^\circ$ **69°** 10. $m\angle 1 = 89^\circ$ **1°** 11. $m\angle 1 = 5^\circ$ **85°**

SUPPLEMENTARY ANGLES $\angle 1$ and $\angle 2$ are supplementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

12. $m\angle 1 = 60^\circ$ **120°** 13. $m\angle 1 = 155^\circ$ **25°** 14. $m\angle 1 = 130^\circ$ **50°** 15. $m\angle 1 = 27^\circ$ **153°**

16. **★ MULTIPLE CHOICE** The arm of a crossing gate moves 37° from vertical. How many more degrees does the arm have to move so that it is horizontal? **B**

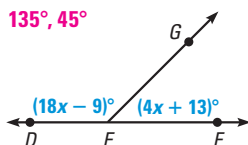
- (A) 37°
(B) 53°
(C) 90°
(D) 143°



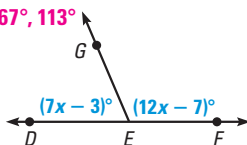
EXAMPLE 3
for Exs. 17–19

xy ALGEBRA Find $m\angle DEG$ and $m\angle GEF$.

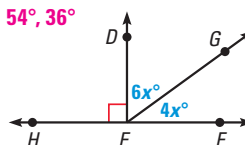
17. **135°, 45°**



18. **67°, 113°**



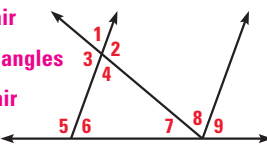
19. **54°, 36°**



EXAMPLE 4
for Exs. 20–27

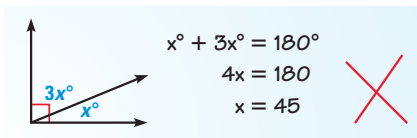
IDENTIFYING ANGLE PAIRS Use the diagram below. Tell whether the angles are **vertical angles**, a **linear pair**, or **neither**.

20. $\angle 1$ and $\angle 4$ **vertical angles**
21. $\angle 1$ and $\angle 2$ **linear pair**
22. $\angle 3$ and $\angle 5$ **vertical angles**
23. $\angle 2$ and $\angle 3$ **vertical angles**
24. $\angle 7$, $\angle 8$, and $\angle 9$ **linear pair**
25. $\angle 5$ and $\angle 6$ **linear pair**
26. $\angle 6$ and $\angle 7$ **neither**
27. $\angle 5$ and $\angle 9$ **neither**



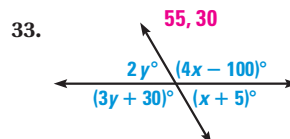
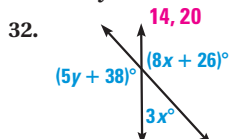
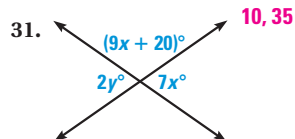
28. **xy ALGEBRA** Two angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle. **36° and 144°**

29. **ERROR ANALYSIS** Describe and correct the error made in finding the value of x . **The angles are complementary so they should be equal to 90° ; $x + 3x = 90^\circ$, $4x = 90^\circ$, $x = 22.5$.**



30. **★ MULTIPLE CHOICE** The measure of one angle is 24° greater than the measure of its complement. What are the measures of the angles? **C**
- (A) 24° and 66° (B) 24° and 156° (C) 33° and 57° (D) 78° and 102°

- B xy ALGEBRA** Find the values of x and y .



Teaching Strategy

Exercises 8–15 To help students remember that measures of complementary angles add up to 90° and those of supplementary angles add up to 180° , observe that C comes before S in the alphabet and 90 is smaller than 180.

Avoiding Common Errors

Exercises 24 Some students may say that these angles form a linear pair. Point out that here we have three angles, not a pair of angles.

Study Strategy

Exercise 30 Check each choice to see if one angle measure is 24° greater than the other and whether the measures add up to 90° . Have students explain why choices B and D can be eliminated without any arithmetic.

Mathematical Reasoning

Exercises 34 and 35 Have students compare the wording in these two exercises. Have them describe in their own words how the answer to one of these exercises implies the answer to the other.

Exercises 46–48 Ask students to explain how they arrived at their answers. Some students may say that they simply looked at the clocks to decide. Ask how they would respond if someone challenged one of their answers.

Internet Reference

Exercises 49–52 More information about the Rock and Roll Hall of Fame can be found at www.rockhall.com/museum

34. Never; an obtuse angle is larger than 90° , and it is not possible to have a complement of an angle that is greater than 90° .

35. Never; a straight angle is 180° , and it is not possible to have a complement of an angle that is 180° .

36. Sometimes; a straight angle does not have a supplement.

37. Always; the sum of complementary angles is 90° , so each angle must be less than 90° , making them acute.

38. Always; an acute angle is less than 90° , since the sum of supplementary angles is 180° and one of the angles is less than 90° , the other angle must be larger than 90° , which makes it obtuse.

45. $(x + 90)^\circ$. **Sample answer:** Since $m\angle GHJ = x^\circ$ and $\angle RST$ is its complement, $m\angle RST = (90 - x)^\circ$. $\angle ABC$ is a supplement of $\angle RST$ so $m\angle ABC = 180^\circ - (90 - x)^\circ$, $m\angle ABC = (x + 90)^\circ$.

REASONING Tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning. 34–38. See margin.

34. An obtuse angle has a complement.

35. A straight angle has a complement.

36. An angle has a supplement.

37. The complement of an acute angle is an acute angle.

38. The supplement of an acute angle is an obtuse angle.

FINDING ANGLES $\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.

39. $m\angle A = (3x + 2)^\circ$
 $m\angle B = (x - 4)^\circ$
 $71^\circ, 19^\circ$

40. $m\angle A = (15x + 3)^\circ$
 $m\angle B = (5x - 13)^\circ$
 $78^\circ, 12^\circ$

41. $m\angle A = (11x + 24)^\circ$
 $m\angle B = (x + 18)^\circ$
 $68^\circ, 22^\circ$

FINDING ANGLES $\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.

42. $m\angle A = (8x + 100)^\circ$
 $m\angle B = (2x + 50)^\circ$
 $124^\circ, 56^\circ$

43. $m\angle A = (2x - 20)^\circ$
 $m\angle B = (3x + 5)^\circ$
 $58^\circ, 122^\circ$

44. $m\angle A = (6x + 72)^\circ$
 $m\angle B = (2x + 28)^\circ$
 $132^\circ, 48^\circ$

C 45. **CHALLENGE** You are given that $\angle GHJ$ is a complement of $\angle RST$ and $\angle RST$ is a supplement of $\angle ABC$. Let $m\angle GHJ$ be x° . What is the measure of $\angle ABC$? Explain your reasoning. See margin.

PROBLEM SOLVING

A **IDENTIFYING ANGLES** Tell whether the two angles shown are complementary, supplementary, or neither.

46.



complementary



47.



neither



48.



supplementary



49–52. Sample answers are given.

49. $\angle FGB$, $\angle BGC$

50. $\angle FGE$, $\angle BGC$

51. $\angle AGE$, $\angle EGD$

52. $\angle AGB$, $\angle BGC$

ARCHITECTURE The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example of the indicated type of angle pair.

49. Supplementary angles

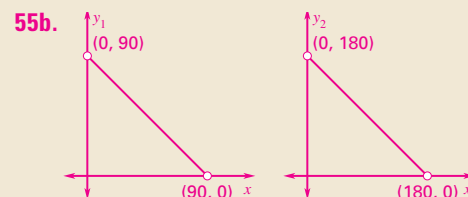
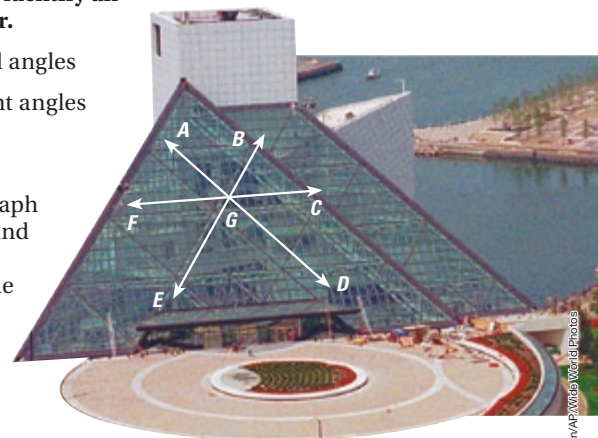
50. Vertical angles

51. Linear pair

52. Adjacent angles

53. **★ SHORT RESPONSE** Use the photograph shown at the right. Given that $\angle FGB$ and $\angle BGC$ are supplementary angles, and $m\angle FGB = 120^\circ$, explain how to find the measure of the complement of $\angle BGC$.

Sample answer: Subtract 90° from $m\angle FGB$.



54. The shadow gets shorter and the angle measure increases.

55a.
 $y_1 = 90 - x$,
 $0 < x < 90$;
 $y_2 = 180 - x$,
 $0 < x < 180$; the
 measure of a
 complement
 must be less
 than 90° and the
 measure of its
 supplement
 must be less
 than 180° .

- B** 54. **SHADOWS** The length of a shadow changes as the sun rises. In the diagram below, the length of \overline{CB} is the length of a shadow. The end of the shadow is the vertex of $\angle ABC$, which is formed by the ground and the sun's rays. Describe how the shadow and angle change as the sun rises.



- 55. ♦ MULTIPLE REPRESENTATIONS** Let x° be an angle measure. Let y_1° be the measure of a complement of the angle and let y_2° be the measure of a supplement of the angle.
- Writing an Equation** Write equations for y_1 as a function of x , and for y_2 as a function of x . What is the domain of each function? Explain.
 - Drawing a Graph** Graph each function and describe its range.
 See margin for art; $0 < y_1 < 90$, $0 < y_2 < 180$.
- 56. CHALLENGE** The sum of the measures of two complementary angles exceeds the difference of their measures by 86° . Find the measure of each angle. Explain how you found the angle measures.
 $47^\circ, 43^\circ$. Sample answer: Solve the system: $x + y = 90$, $x + y - 86 = x - y$.

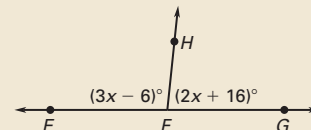
C

5 ASSESS AND RETEACH

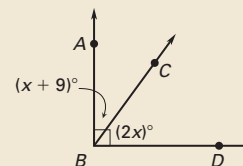
Daily Homework Quiz

Also available online

- $\angle 1$ and $\angle 2$ are supplementary. If $m\angle 1 = 97^\circ$, find $m\angle 2$. 83°
- $\angle 3$ and $\angle 4$ are complementary angles. If $m\angle 3 = 74^\circ$, find $m\angle 4$. 16°
- Find $m\angle EFH$. 96°



- Find $m\angle ABC$. 36°



- Is it possible to draw a figure that contains exactly one pair of vertical angles? Explain. No; once you have drawn a pair of vertical angles, you have drawn two pairs of opposite rays. This automatically gives another pair of vertical angles.



Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

See **EXTRA PRACTICE** in Student Resources



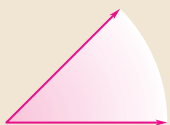
ONLINE QUIZ at my.hrw.com

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Draw an acute angle and shade the interior.



2. Find the measure of the supplement of a 130° angle. 50°
3. Find the measure of the complement of an 86° angle. 4°

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with previous lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you classify polygons?

Tell students they will learn how to answer this question by learning the names of the most common polygons.

READ VOCABULARY

A *plane figure* is two-dimensional. Later, you will study three-dimensional *space figures* such as prisms and cylinders.

1.6 Classify Polygons



Before

You classified angles.

Now

You will classify polygons.

Why?

So you can find lengths in a floor plan, as in Ex. 32.

Key Vocabulary

- **polygon**
side, vertex
- **convex**
- **concave**
- **n -gon**
- **equilateral**
- **equiangular**
- **regular**

KEY CONCEPT

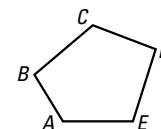
For Your Notebook

Identifying Polygons

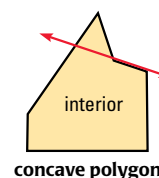
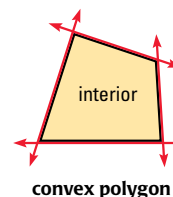
In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is *vertices*. A polygon can be named by listing the vertices in consecutive order. For example, $ABCDE$ and $CDEAB$ are both correct names for the polygon at the right.



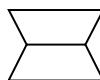
A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called *nonconvex* or **concave**.



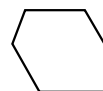
EXAMPLE 1 Identify polygons

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

a.



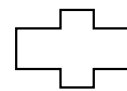
b.



c.



d.



Solution

- Some segments intersect more than two segments, so it is not a polygon.
- The figure is a convex polygon.
- Part of the figure is not a segment, so it is not a polygon.
- The figure is a concave polygon.

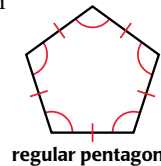
CLASSIFYING POLYGONS A polygon is named by the number of its sides.

Number of sides	Type of polygon	Number of sides	Type of polygon
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	n	n -gon

The term **n -gon**, where n is the number of a polygon's sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent.

In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.



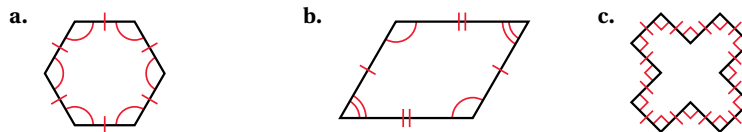
regular pentagon

EXAMPLE 2 Classify polygons

READ DIAGRAMS

Double marks are used in part (b) of Example 2 to show that more than one pair of sides are congruent and more than one pair of angles are congruent.

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.



Solution

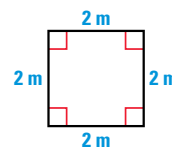
- The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.
- The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.
- The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

Animated Geometry at my.hrw.com

GUIDED PRACTICE for Examples 1 and 2

- Sketch an example of a convex heptagon and an example of a concave heptagon. **See margin.**
- Classify the polygon shown at the right by the number of sides. *Explain* how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.

Quadrilateral; they all have the same length; they are all right angles.



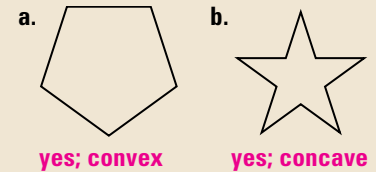
Motivating the Lesson

Ask students to give examples of triangles, squares, rectangles, and other figures in the real world.

3 TEACH

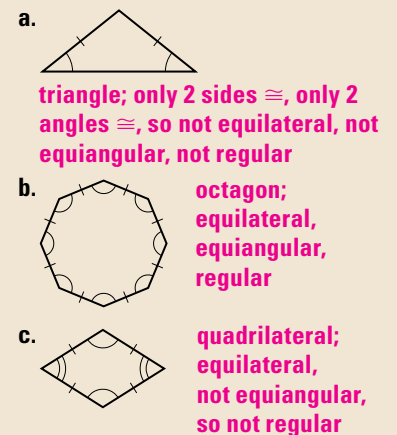
Extra Example 1

Tell whether each figure is a polygon. If it is, tell whether it is convex or concave.



Extra Example 2

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.



Animated Geometry
my.hrw.com

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

1. See Additional Answers.

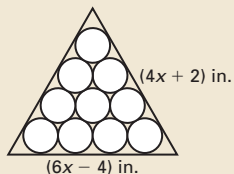
Differentiated Instruction

Below Level Tell the students that a polygon is concave if it is dented inward at one or more of its vertices. To help them remember this term, you may want to suggest that they make a mental connection with the words *cavity* and *cave*.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3

A rack for billiard balls is shaped like an equilateral triangle. Find the length of a side. **14 in.**



Key Question to Ask for Example 3

- What is the distance around the table? **180 in.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you classify polygons?

- A polygon is a plane figure formed by three or more line segments. Each side intersects exactly two other sides, one at each endpoint, so that no two sides with a common endpoint are collinear.
- A polygon is convex if no line that contains a side of the polygon contains a point in the interior. Otherwise, the polygon is concave.
- A polygon is regular if all sides are congruent and all angles in the interior are congruent.

The most basic way of classifying a polygon is by the number of sides. You can also tell whether the polygon is convex or concave, or indicate whether all the sides or angles are congruent. If all the sides of a convex polygon are congruent and all the angles are congruent, the polygon is a regular polygon.

2. Yes, the string will match the sides of a convex polygon, so it will be the perimeter of the polygon; no, the length of the string will be less than the perimeter of the polygon.

3. polygon; concave

4. Not a polygon; part of the figure is not a segment.

5. polygon; convex

6. Not a polygon; some segments intersect more than two segments.

EXAMPLE 3 Find side lengths

READ VOCABULARY
Hexagonal means
"shaped like a hexagon."

xy ALGEBRA A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.

$(3x + 6)$ in.

$(4x - 2)$ in.

Solution

First, write and solve an equation to find the value of x . Use the fact that the sides of a regular hexagon are congruent.

$$3x + 6 = 4x - 2 \quad \text{Write equation.}$$

$$6 = x - 2 \quad \text{Subtract } 3x \text{ from each side.}$$

$$8 = x \quad \text{Add 2 to each side.}$$

Then find a side length. Evaluate one of the expressions when $x = 8$.

$$3x + 6 = 3(8) + 6 = 30$$

- The length of a side of the table is 30 inches.



GUIDED PRACTICE for Example 3

3. The expressions $8y^\circ$ and $(9y - 15)^\circ$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle. **120°**

1.6 EXERCISES

HOMEWORK KEY

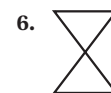
- = See **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 33
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 37, 39, and 40

SKILL PRACTICE

- A** 1. **VOCABULARY** Explain what is meant by the term n -gon.
An n -gon is a polygon with n sides.
2. ★ **WRITING** Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? Explain. **See margin.**

EXAMPLE 1
for Exs. 3–7

IDENTIFYING POLYGONS Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is *convex* or *concave*. **3–6. See margin.**

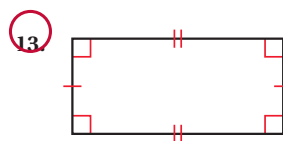
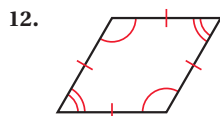
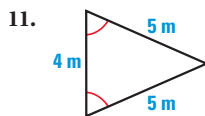
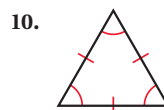
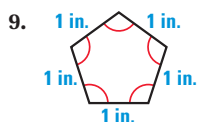
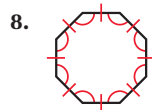


7. ★ **MULTIPLE CHOICE** Which of the figures is a concave polygon? **C**



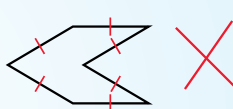
EXAMPLE 2
for Exs. 8–14

CLASSIFYING Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. *Explain your reasoning.* 8–13. See margin.

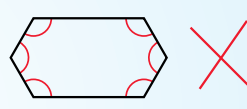


14. **ERROR ANALYSIS** Two students were asked to draw a regular hexagon, as shown below. Describe the error made by each student.
Student A: the hexagon is concave, Student B: the hexagon does not have congruent sides.

Student A



Student B



15. **xy ALGEBRA** The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon. **8 in.**
16. **xy ALGEBRA** The expressions $(9x + 5)^\circ$ and $(11x - 25)^\circ$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon. **140°**
17. **xy ALGEBRA** The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side. **3 ft**

B

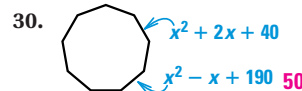
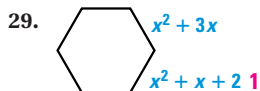
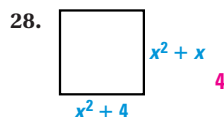
USING PROPERTIES Tell whether the statement is *always*, *sometimes*, or *never* true.

- | | |
|---|--|
| 18. A triangle is convex. always | 19. A decagon is regular. sometimes |
| 20. A regular polygon is equiangular. always | 21. A circle is a polygon. never |
| 22. A polygon is a plane figure. always | 23. A concave polygon is regular. never |
- 24–27. See margin.**

DRAWING Draw a figure that fits the description.

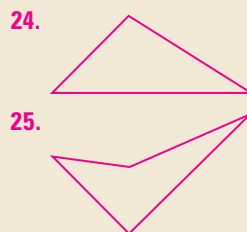
24. A triangle that is not regular
25. A concave quadrilateral
26. A pentagon that is equilateral but not equiangular
27. An octagon that is equiangular but not equilateral

xy ALGEBRA Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of x .



8. Octagon; regular; it has 8 congruent sides and angles.
9. Pentagon; regular; it has 5 congruent sides and angles.
10. Triangle; regular; it has 3 congruent sides and angles.
11. Triangle; none of these; the sides and/or the angles are not all congruent.
12. Quadrilateral; equilateral; it has 4 congruent sides.
13. Quadrilateral; equiangular; it has 4 congruent angles.

24–27. Sample answers are given.



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:

Exs. 1–10, 14–21, 24, 25, 32–40

Average:

Day 1:

Exs. 1, 2, 4–7, 9–11, 14–17, 21–30, 32–41

Advanced:

Day 1:

Exs. 1, 2, 5–7, 12, 13, 15–17, 22–32*, 35–42*

Block:

Exs. 1, 2, 4–7, 9–11, 14–17, 21–30, 32–41 (with the previous lesson)

Differentiated Instruction

See *Differentiated Instruction*

Resources for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 15, 34, 39

Average: 5, 10, 16, 35, 39

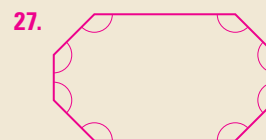
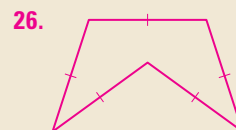
Advanced: 6, 12, 17, 36, 39

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.



Avoiding Common Errors

Exercise 11 Some students may classify the triangle as equilateral or equiangular because two sides or two angles are congruent, respectively. Remind students that *all* the sides or *all* the angles must be congruent for a figure to be equilateral or equiangular.

Teaching Strategy

Exercise 37 Have the students graph the given ordered pairs and use reasoning to determine the possible locations of the other two vertices.

31. $72^\circ, 72^\circ, 36^\circ$. **Sample answer:** To find the measure of the angles of the pentagon, solve $20x + 48 = 33x + 9$ to get $x = 3$. This will get the measures of the angles of the pentagon equal to 108° . $\angle ABC$ and $\angle ACB$ form a linear pair with an angle of a pentagon so they are both equal to $180^\circ - 108^\circ = 72^\circ$. $\angle BAC, \angle CAD$, and $\angle DAE$ must have a sum of 180° . Since $\angle CAB \cong \angle DAE$, $2y + 108 = 180$, $y = 36$, so $m\angle CAB = 36^\circ$.

32b. Octagon, it appears to be regular.

EXAMPLE 2 for Exs. 33–36

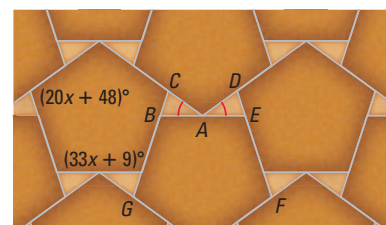
33. triangle; regular

34. quadrilateral; equiangular

35. octagon; regular

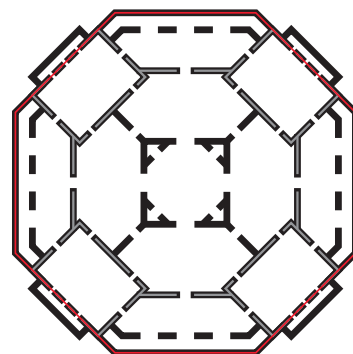
36. dodecagon; none of these

- C** 31. **CHALLENGE** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain your reasoning. See margin.*



PROBLEM SOLVING

- A** 32. **ARCHITECTURE** Longwood House, shown at the beginning of this lesson, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.
- Tell whether the red polygon in the diagram is *convex* or *concave*. **convex**
 - Classify the red polygon and tell whether it appears to be regular.



SIGNS Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral*, *equiangular*, or *regular*.

33.



34.



35.



36.



37. **★ MULTIPLE CHOICE** Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, -4)$. Which of the following could be the other two vertices? **C**

(A) $C(4, 4)$ and $D(4, -4)$

(B) $C(-4, 4)$ and $D(-4, -4)$

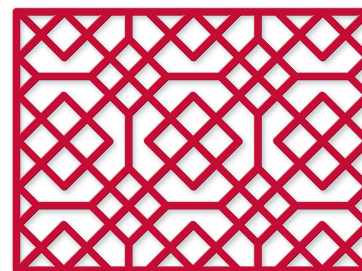
(C) $C(8, -4)$ and $D(8, 4)$

(D) $C(0, 8)$ and $D(0, -8)$

38. **MULTI-STEP PROBLEM** The diagram shows the design of a lattice made in China in 1850.

38a–b. **See margin.**

- Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.
- Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.



38a–b. **Sample:**



triangle
convex
none



quadrilateral
convex
regular



pentagon
convex
none



heptagon
concave
none



octagon
concave
none

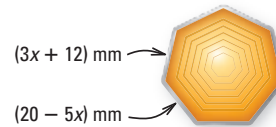
EXAMPLE 3

for Ex. 39

39. 105 mm; each side of the button is 15 millimeters long, so the perimeter of the button is $15(7) = 105$ millimeters.

40a. See margin for art. Sample answer: The difference between successive number of diagonals is increasing by one.

39. ★ **SHORT RESPONSE** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain.*



40. ★ **EXTENDED RESPONSE** A segment that joins two nonconsecutive vertices of a polygon is called a *diagonal*. For example, a quadrilateral has two diagonals, as shown below.

Type of polygon	Diagram	Number of sides	Number of diagonals
Quadrilateral		4	2
Pentagon	?	? 5	? 5
Hexagon	?	? 6	? 9
Heptagon	?	? 7	? 14

- a. Copy and complete the table. *Describe* any patterns you see.
 b. How many diagonals does an octagon have? a nonagon? *Explain.*
 c. The expression $\frac{n(n-3)}{2}$ can be used to find the number of diagonals in an n -gon. Find the number of diagonals in a 60-gon. **20 diagonals; 27 diagonals. Sample answer: The pattern described continues.**

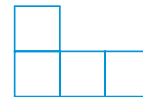
41. **LINE SYMMETRY** A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.



regular quadrilateral
4 lines of symmetry

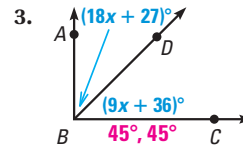
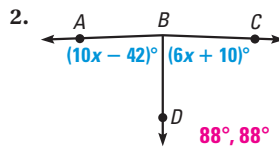
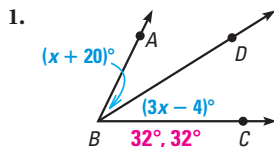
- a. A regular triangle **3** b. A regular pentagon **5**
 c. A regular hexagon **6** d. A regular octagon **8**

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge. **See margin.**



QUIZ

In each diagram, \overrightarrow{BD} bisects $\angle ABC$. Find $m\angle ABD$ and $m\angle DBC$.



Find the measure of (a) the complement and (b) the supplement of $\angle 1$.

4. $m\angle 1 = 47^\circ$ 5. $m\angle 1 = 19^\circ$ 6. $m\angle 1 = 75^\circ$ 7. $m\angle 1 = 2^\circ$

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*.



See **EXTRA PRACTICE** in Student Resources



ONLINE QUIZ at my.hrw.com

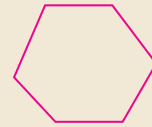
47

5 ASSESS AND RETEACH

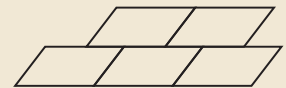
Daily Homework Quiz

Also available online

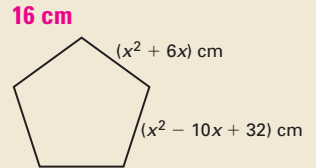
1. Draw a convex hexagon.



2. This figure shows the tiles on a kitchen floor. What type of polygon are the tiles? Are they regular polygons? **quadrilaterals; not regular**



3. This figure is a regular polygon. Find the length of each side.



Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.



42. See Additional Answers.

Alternative Strategy

Example 2 in the previous lesson was solved by relying on the markings on the geometric figures. In this example, students measure the figures using informal tools.

Teaching Strategy

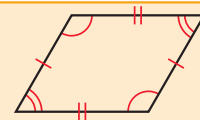
Students are generally discouraged from relying on the appearance of congruence or similarity of figures, but this method demonstrates that it is sometimes appropriate to take your own measurements to solve problems.

Another Way to Solve Example 2

MULTIPLE REPRESENTATIONS In Example 4, you saw how to use markings on geometric figures to determine whether sides and angles are congruent. *Using measuring tools* can allow you to decide whether measures in a diagram are equal.

PROBLEM

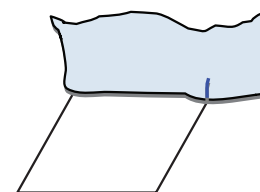
Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.



METHOD 1

Using a Measuring Tool For problems in your Geometry book, you are expected to rely on the markings on the diagrams to determine whether sides and angles are congruent, but you can also use a piece of paper to test whether lengths and angles in a diagram are equal.

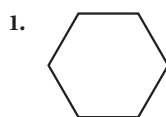
STEP 1 Choose a Tool to Measure Length Because it can be hard to use a ruler on a diagram in your book, you can copy the diagram onto a piece of paper. Or you can use a piece of paper to measure lengths by marking a side length on a scrap of paper and then comparing to the other sides.



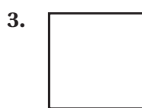
STEP 2 Choose a Tool to Measure Angle Copy the diagram onto a piece of paper so you can use a protractor. Or you can use a piece of paper to measure angles by folding a scrap of paper to match an angle measure and then comparing to the other angles.

PRACTICE

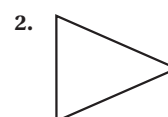
Choose a tool to use to measure the lengths and angles of each diagram. Tell whether the figure is *equilateral*, *equiangular*, or *regular*.



regular



regular



none of these



equiangular

HOUGHTON MIFFLIN HARCOURT  HO
ON MIFFLIN HARCOURT  HOUGHTON M
RT  HOUGHTON MIFFLIN HARCOURT
HOUGHTON MIFFLIN HARCOURT  HO
IN HARCOURT  HOUGHTON MIFFLIN H
HOUGHTON MIFFLIN HARCOURT  HO
ON MIFFLIN HARCOURT  HOUGHTON M
RT  HOUGHTON MIFFLIN HARCOURT
HOUGHTON MIFFLIN HARCOURT  HO

MIXED REVIEW of Problem Solving

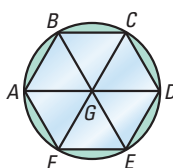
3a. **Sample answer:** $\angle DGB$ and $\angle BGH$, $\angle HGC$ and $\angle CGF$, $\angle DBG$ and $\angle HBG$, $\angle HCG$ and $\angle FCG$, $\angle FCG$, $\angle HGD$ and $\angle HGF$

3b. 21° , $\angle FGC \cong \angle DGB$; 69° , $\angle BGH$ is complementary to $\angle DGB$; 69° , $\angle HGC$ is complementary to $\angle FGC$.

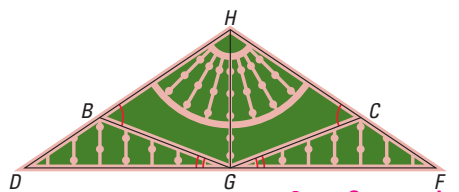
3c. 55° , $\angle HCG \cong \angle HBG$; 125° , $\angle DBG$ is supplementary to $\angle HBG$; 125° , $\angle FCG$ is supplementary to $\angle HCG$.

5a. 108 bricks, about 98 bricks; convert total perimeter to inches and then divide by 10.

- MULTI-STEP PROBLEM** You are covering the rectangular roof of a shed with shingles. The roof is a rectangle that is 4 yards long and 3 yards wide. Asphalt shingles cost \$.75 per square foot and wood shingles cost \$1.15 per square foot.
 - Find the area of the roof in square feet. **108 ft²**
 - Find the cost of using asphalt shingles and the cost of using wood shingles. **\$81, \$124.20**
 - About how much more will you pay to use wood shingles for the roof? **\$43.20**
- OPEN-ENDED** In the window below, name a convex polygon and a concave polygon. Classify each of your polygons by the number of sides. **Sample answer:** *GBC, GBCDEFA*; triangle, heptagon



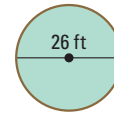
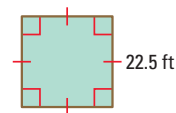
- EXTENDED RESPONSE** The diagram shows a decoration on a house. In the diagram, $\angle HGD$ and $\angle HGF$ are right angles, $m\angle DGB = 21^\circ$, $m\angle HBG = 55^\circ$, $\angle DGB \cong \angle FGC$, and $\angle HBG \cong \angle HCG$.



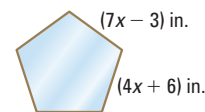
3a–c. See margin.

- List two pairs of complementary angles and five pairs of supplementary angles.
 - Find $m\angle FGC$, $m\angle BGH$, and $m\angle HGC$. Explain your reasoning.
 - Find $m\angle HCG$, $m\angle DBG$, and $m\angle FCG$. Explain your reasoning.
- GRIDDED ANSWER** $\angle 1$ and $\angle 2$ are supplementary angles, and $\angle 1$ and $\angle 3$ are complementary angles. Given $m\angle 1$ is 28° less than $m\angle 2$, find $m\angle 3$ in degrees. **14°**

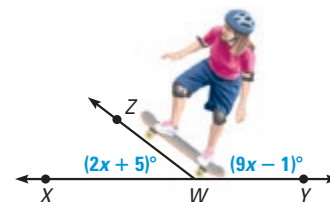
- EXTENDED RESPONSE** You use bricks to outline the borders of the two gardens shown below. Each brick is 10 inches long.



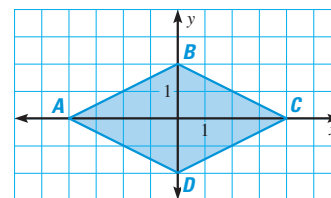
- You lay the bricks end-to-end around the border of each garden. How many bricks do you need for each garden? Explain. **See margin.**
 - The bricks are sold in bundles of 100. How many bundles should you buy? Explain. **3 bundles; 206 bricks are needed, thus 3 bundles.**
- SHORT RESPONSE** The frame of a mirror is a regular pentagon made from pieces of bamboo. Use the diagram to find how many feet of bamboo are used in the frame. **7.5 ft of bamboo**



- GRIDDED ANSWER** As shown in the diagram, a skateboarder tilts one end of a skateboard. Find $m\angle ZWX$ in degrees. **37°**



- SHORT RESPONSE** Use the diagram below.



8√5 units

- Find the perimeter of quadrilateral ABCD.
- Find the area of triangle ABC and the area of triangle ADC. What is the area of quadrilateral ABCD? Explain. **8 square units, 8 square units; 16 square units; the area of the quadrilateral is the sum of the areas of the two triangles.**

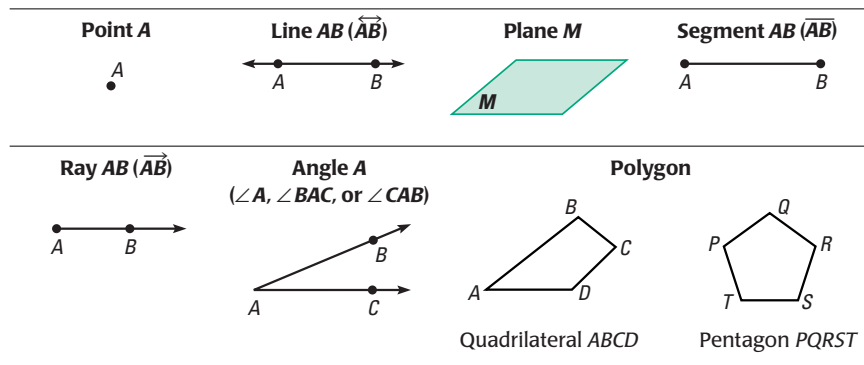
BIG IDEAS

For Your Notebook

Big Idea 1

Describing Geometric Figures

You learned to identify and classify geometric figures.



Big Idea 2

Measuring Geometric Figures

SEGMENTS You measured segments in the coordinate plane.

Distance Formula

Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$:

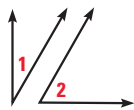
$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint Formula

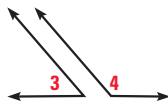
Coordinates of midpoint M of \overline{AB} , with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

ANGLES You classified angles and found their measures.

**Complementary angles**

$$m\angle 1 + m\angle 2 = 90^\circ$$

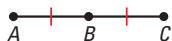
**Supplementary angles**

$$m\angle 3 + m\angle 4 = 180^\circ$$

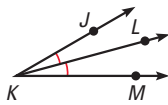
Big Idea 3

Understanding Equality and Congruence

Congruent segments have equal lengths. Congruent angles have equal measures.



$$\overline{AB} \cong \overline{BC} \text{ and } AB = BC$$



$$\angle JKL \cong \angle LKM \text{ and } m\angle JKL = m\angle LKM$$

Additional Resources

The following resources are available to help review the materials in this chapter.

Chapter Resource Book

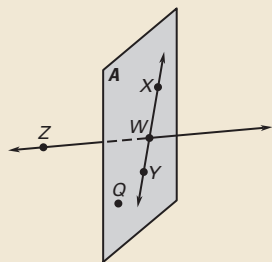
- Chapter Review Games and Activities
- Cumulative Practice

Student Resources in Spanish**@HomeTutor****Vocabulary Practice**

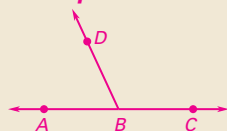
Vocabulary practice is available at my.hrw.com

Extra Example 1

Use the diagram shown.



- a. Give another name for \overleftrightarrow{XY} .
 \overleftrightarrow{XW} , \overleftrightarrow{YW} , \overleftrightarrow{WX} , \overleftrightarrow{WY} , or \overleftrightarrow{YX}
- b. Name three points that are collinear. X, W, Y
- c. Name four points that are coplanar. X, W, Y, Q

2. Sample:**REVIEW KEY VOCABULARY**

For a list of postulates and theorems, see p. PT2.

- undefined terms
point, line, plane
- collinear, coplanar points
- defined terms
- line segment, endpoints
- ray, opposite rays
- intersection
- postulate, axiom
- coordinate
- distance
- between
- congruent segments
- midpoint
- segment bisector
- angle
sides, vertex, measure
- acute, right, obtuse, straight
- congruent angles
- angle bisector
- construction
- complementary angles
- supplementary angles
- adjacent angles
- linear pair
- vertical angles
- polygon
side, vertex
- convex, concave
- n -gon
- equilateral, equiangular, regular

VOCABULARY EXERCISES

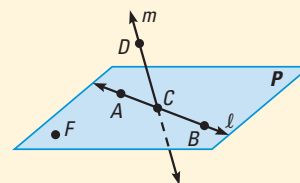
- Copy and complete: Points A and B are the ? of \overline{AB} . **endpoints**
- Draw an example of a *linear pair*. **See margin.**
- If Q is between points P and R on \overleftrightarrow{PR} , and $PQ = QR$, then Q is the ? of \overline{PR} . **midpoint**

REVIEW EXAMPLES AND EXERCISES

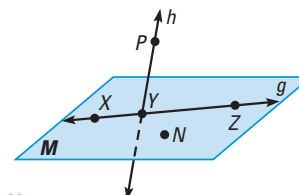
Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

1.1**Identify Points, Lines, and Planes****EXAMPLE**

Use the diagram shown at the right.

Another name for \overleftrightarrow{CD} is line m .Points A, B , and C are collinear.Points A, B, C , and F are coplanar.**EXERCISES**

- Give another name for line g . **Sample answer: \overleftrightarrow{YZ}**
- Name three points that are *not* collinear.
Sample answer: points P, Y, Z
- Name four points that are coplanar.
Sample answer: N, X, Y, Z
- Name a pair of opposite rays. **$\overrightarrow{YZ}, \overrightarrow{YX}$**
- Name the intersection of line h and plane M . **point Y**



EXAMPLES 1, 2, and 3
for Exs. 4–8

1.2 Use Segments and Congruence

EXAMPLE

Find the length of \overline{HJ} .

$$GJ = GH + HJ \quad \text{Segment Addition Postulate}$$

$$27 = 18 + HJ \quad \text{Substitute 27 for } GJ \text{ and 18 for } GH.$$

$$9 = HJ \quad \text{Subtract 18 from each side.}$$



EXERCISES

Find the indicated length.

9. Find AB . **1.2**



10. Find NP . **30**



11. Find XY . **7**



12. The endpoints of \overline{DE} are $D(-4, 11)$ and $E(-4, -13)$. The endpoints of \overline{GH} are $G(-14, 5)$ and $H(-9, 5)$. Are \overline{DE} and \overline{GH} congruent? *Explain.* **no; $DE = 24$, $GH = 5$**

1.3 Use Midpoint and Distance Formulas

EXAMPLE

\overline{EF} has endpoints $E(1, 4)$ and $F(3, 2)$. Find (a) the length of \overline{EF} rounded to the nearest tenth of a unit, and (b) the coordinates of the midpoint M of \overline{EF} .

- a. Use the Distance Formula.

$$EF = \sqrt{(3 - 1)^2 + (2 - 4)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} \approx 2.8 \text{ units}$$

- b. Use the Midpoint Formula.

$$M\left(\frac{1 + 3}{2}, \frac{4 + 2}{2}\right) = M(2, 3)$$

EXERCISES

13. Point M is the midpoint of \overline{JK} . Find JK when $JM = 6x - 7$ and $MK = 2x + 3$. **16**

In Exercises 14–17, the endpoints of a segment are given. Find the length of the segment rounded to the nearest tenth. Then find the coordinates of the midpoint of the segment.

14. $A(2, 5)$ and $B(4, 3)$ **2.8; (3, 4)**

15. $F(1, 7)$ and $G(6, 0)$ **8.6; (3.5, 3.5)**

16. $H(-3, 9)$ and $J(5, 4)$ **9.4; (1, 6.5)**

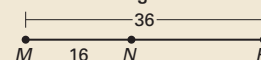
17. $K(10, 6)$ and $L(0, -7)$ **16.4; (5, -0.5)**

18. Point $C(3, 8)$ is the midpoint of \overline{AB} . One endpoint is $A(-1, 5)$. Find the coordinates of endpoint B . **(7, 11)**

19. The endpoints of \overline{EF} are $E(2, 3)$ and $F(8, 11)$. The midpoint of \overline{EF} is M . Find the length of \overline{EM} . **5**

Extra Example 2

Find the length of \overline{NP} . **20**



Extra Example 3

\overline{PQ} has endpoints $P(-4, -2)$ and $Q(-6, 8)$. Find

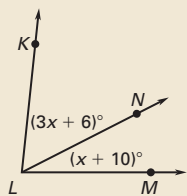
- a. the length of \overline{PQ} to the nearest tenth of a unit. **10.2 units**
b. the midpoint M of \overline{PQ} . **(-5, 3)**

EXAMPLES
2, 3, and 4
for Exs. 9–12

EXAMPLES
2, 3, and 4
for Exs. 13–19

Extra Example 4

Given that $m\angle KLM$ is 84° , find $m\angle KLN$ and $m\angle NLM$. **$57^\circ, 27^\circ$**

**Extra Example 5**

- a. $\angle 1$ and $\angle 2$ are complementary angles. Given that $m\angle 1 = 59^\circ$, find $m\angle 2$. **31°**
- b. $\angle 3$ and $\angle 4$ are supplementary angles. Given that $m\angle 3 = 84^\circ$, find $m\angle 4$. **96°**

EXAMPLES
3 and 5
for Exs. 20–21

1.4 Measure and Classify Angles**EXAMPLE**

Given that $m\angle YXV$ is 60° , find $m\angle YXZ$ and $m\angle ZXV$.

STEP 1 Find the value of x .

$$m\angle YXV = m\angle YXZ + m\angle ZXV$$

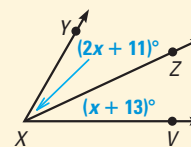
$$60^\circ = (2x + 11)^\circ + (x + 13)^\circ$$

$$x = 12$$

STEP 2 Evaluate the given expressions when $x = 12$.

$$m\angle YXZ = (2x + 11)^\circ = (2 \cdot 12 + 11)^\circ = 35^\circ$$

$$m\angle ZXV = (x + 13)^\circ = (12 + 13)^\circ = 25^\circ$$



Angle Addition Postulate

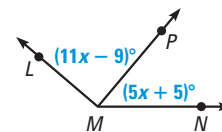
Substitute angle measures.

Solve for x .

EXERCISES

20. In the diagram shown at the right, $m\angle LMN = 140^\circ$. Find $m\angle PMN$. **50°**

21. \overrightarrow{VZ} bisects $\angle UVW$, and $m\angle UVZ = 81^\circ$. Find $m\angle UVW$. Then classify $\angle UVW$ by its angle measure. **162° ; obtuse**

**1.5 Describe Angle Pair Relationships****EXAMPLE**

- a. $\angle 1$ and $\angle 2$ are complementary angles. Given that $m\angle 1 = 37^\circ$, find $m\angle 2$.

$$m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 37^\circ = 53^\circ$$

- b. $\angle 3$ and $\angle 4$ are supplementary angles. Given that $m\angle 3 = 106^\circ$, find $m\angle 4$.

$$m\angle 4 = 180^\circ - m\angle 3 = 180^\circ - 106^\circ = 74^\circ$$

EXERCISES

$\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

22. $m\angle 1 = 12^\circ$ **78°** 23. $m\angle 1 = 83^\circ$ **7°** 24. $m\angle 1 = 46^\circ$ **44°** 25. $m\angle 1 = 2^\circ$ **88°**

$\angle 3$ and $\angle 4$ are supplementary angles. Given the measure of $\angle 3$, find $m\angle 4$.

26. $m\angle 3 = 116^\circ$ **64°** 27. $m\angle 3 = 56^\circ$ **124°** 28. $m\angle 3 = 89^\circ$ **91°** 29. $m\angle 3 = 12^\circ$ **168°**

30. $\angle 1$ and $\angle 2$ are complementary angles. Find the measures of the angles when $m\angle 1 = (x - 10)^\circ$ and $m\angle 2 = (2x + 40)^\circ$. **$10^\circ, 80^\circ$**

31. $\angle 1$ and $\angle 2$ are supplementary angles. Find the measures of the angles when $m\angle 1 = (3x + 50)^\circ$ and $m\angle 2 = (4x + 32)^\circ$. Then classify $\angle 1$ by its angle measure. **$92^\circ, 88^\circ$; obtuse**

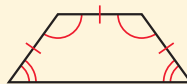
EXAMPLES
2 and 3
for Exs. 22–31

1.6 Classify Polygons

EXAMPLE

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain.*

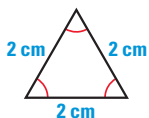
The polygon has four sides, so it is a quadrilateral. It is not equiangular or equilateral, so it is not regular.



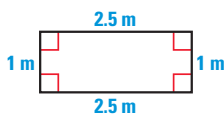
EXERCISES

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain.* 32–34. See margin.

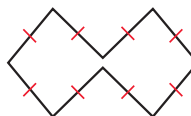
32.



33.



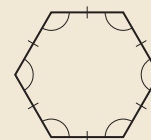
34.



35. Pentagon $ABCDE$ is a regular polygon. The length of \overline{BC} is represented by the expression $5x - 4$. The length of \overline{DE} is represented by the expression $2x + 11$. Find the length of \overline{AB} . 21

Extra Example 6

Classify the polygon shown by the number of sides and decide whether it is a regular polygon.



hexagon, regular

Additional Resources

Assessment Book

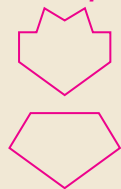
- Chapter Test, Levels A, B, C
- Standardized Chapter Test
- SAT/ACT Chapter Test
- Alternative Assessment

ExamView™ Assessment Suite

Chapter Test

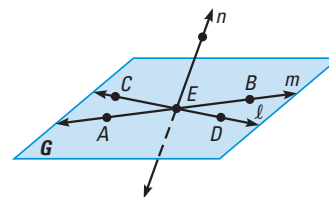
Easily-readable reduced copies of Chapter Test B, the Standardized Chapter Test, and the Alternative Assessment from the Assessment Book can be found at the beginning of this chapter.

20. Sample:



Use the diagram to decide whether the statement is *true* or *false*.

1. Point A lies on line m . **true**
2. Point D lies on line n . **false**
3. Points B, C, E, and Q are coplanar. **false**
4. Points C, E, and B are collinear. **false**
5. Another name for plane G is plane QEC. **false**

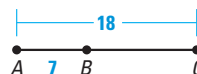


Find the indicated length.

6. Find HJ . **22**



7. Find BC . **11**



8. Find XZ . **71**

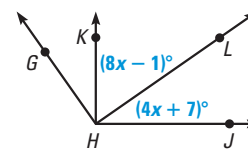


In Exercises 9–11, find the distance between the two points.

9. $T(3, 4)$ and $W(2, 7)$ **$\sqrt{10} \approx 3.2$**
10. $C(5, 10)$ and $D(6, -1)$ **$\sqrt{122} \approx 11.0$**
11. $M(-8, 0)$ and $N(-1, 3)$ **$\sqrt{58} \approx 7.6$**
12. The midpoint of \overline{AB} is $M(9, 7)$. One endpoint is $A(3, 9)$. Find the coordinates of endpoint B. **$(15, 5)$**
13. Line t bisects \overline{CD} at point M, $CM = 3x$, and $MD = 27$. Find CD . **54**

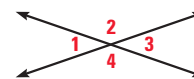
In Exercises 14 and 15, use the diagram.

14. Trace the diagram and extend the rays. Use a protractor to measure $\angle GHJ$. Classify it as *acute*, *obtuse*, *right*, or *straight*. **125° ; obtuse**
15. Given $m\angle KHJ = 90^\circ$, find $m\angle LHJ$. **35°**



16. The measure of $\angle QRT$ is 154° , and \overrightarrow{RS} bisects $\angle QRT$. What are the measures of $\angle QRS$ and $\angle SRT$? **$77^\circ, 77^\circ$**

In Exercises 17 and 18, use the diagram at the right.



17. $\angle 1$ and $\angle 2$,
 $\angle 2$ and $\angle 3$, $\angle 3$
and $\angle 4$, $\angle 4$ and
 $\angle 1$

17. Name four linear pairs.
18. Name two pairs of vertical angles. **$\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$**
19. The measure of an angle is 64° . What is the measure of its complement? What is the measure of its supplement? **$26^\circ, 116^\circ$**
20. A convex polygon has half as many sides as a concave 10-gon. Draw the concave polygon and the convex polygon. Classify the convex polygon by the number of sides it has. **See margin for art; pentagon.**

SOLVE LINEAR EQUATIONS AND WORD PROBLEMS



EXAMPLE 1 Solve linear equations

Solve the equation $-3(x + 5) + 4x = 25$.

$$-3(x + 5) + 4x = 25 \quad \text{Write original equation.}$$

$$-3x - 15 + 4x = 25 \quad \text{Use the Distributive Property.}$$

$$x - 15 = 25 \quad \text{Group and combine like terms.}$$

$$x = 40 \quad \text{Add 15 to each side.}$$



EXAMPLE 2 Solve a real-world problem

MEMBERSHIP COSTS A health club charges an initiation fee of \$50. Members then pay \$45 per month. You have \$400 to spend on a health club membership. For how many months can you afford to be a member?

Let n represent the number of months you can pay for a membership.

$$\$400 = \text{Initiation fee} + (\text{Monthly Rate} \times \text{Number of Months})$$

$$400 = 50 + 45n \quad \text{Substitute.}$$

$$350 = 45n \quad \text{Subtract 50 from each side.}$$

$$7.8 = n \quad \text{Divide each side by 45.}$$

► You can afford to be a member at the health club for 7 months.

EXERCISES

EXAMPLE 1

for Exs. 1–9

Solve the equation.

1. $9y + 1 - y = 49$ **6**

2. $5z + 7 + z = -8$ **$-2\frac{1}{2}$**

3. $-4(2 - t) = -16$ **-2**

4. $7a - 2(a - 1) = 17$ **3**

5. $\frac{4x}{3} + 2(3 - x) = 5$ **$1\frac{1}{2}$**

6. $\frac{2x - 5}{7} = 4$ **$16\frac{1}{2}$**

7. $9c - 11 = -c + 29$ **4**

8. $2(0.3r + 1) = 23 - 0.1r$ **30**

9. $5(k + 2) = 3(k - 4)$ **-11**

EXAMPLE 2

for Exs. 10–12

10. **GIFT CERTIFICATE** You have a \$50 gift certificate at a store. You want to buy a book that costs \$8.99 and boxes of stationery for your friends. Each box costs \$4.59. How many boxes can you buy with your gift certificate? **8 boxes**

11. **CATERING** It costs \$350 to rent a room for a party. You also want to hire a caterer. The caterer charges \$8.75 per person. How many people can come to the party if you have \$500 to spend on the room and the caterer? **17 people**

12. **JEWELRY** You are making a necklace out of glass beads. You use one bead that is $1\frac{1}{2}$ inches long and smaller beads that are each $\frac{3}{4}$ inch long. The necklace is 18 inches long. How many smaller beads do you need? **22 smaller beads**

Extra Example 1

Solve the equation $4(x - 2) + 5x = 19$. **3**

Extra Example 2

Mrs. Clarett ordered business cards. The company charged \$25 for the design and \$20 for each box of cards. If the total charge was \$145, how many boxes of cards did Mrs. Clarett order? **6 boxes**

Using Rubrics

The rubric given on the pupil page is a sample of a three-level rubric. Other rubrics may contain four, five, or six levels. For more information on rubrics, see the *Differentiated Instruction Resources*.

Test-Taking Strategy

Read the question carefully. Watch for different units in the problem. Here the cost is per square foot but the dimensions are in yards. Draw and label a diagram of the dance floor and convert the dimensions to feet. This will prevent conversion errors later in the solution.

Avoiding Common Errors

Students may change square yards to square feet by multiplying by 3. Remind them that there are 9 square feet in one square yard.

Study Strategy

Have students make a detailed list of the steps they would use to solve the problem in Practice. This will help them identify errors and things overlooked in each of the proposed solutions.

Scoring Rubric

Full Credit

- solution is complete and correct

Partial Credit

- solution is complete but has errors, or
- solution is without error but incomplete

No Credit

- no solution is given, or
- solution makes no sense

SHORT RESPONSE QUESTIONS

PROBLEM

You want to rent portable flooring to set up a dance floor for a party. The table below shows the cost of renting portable flooring from a local company. You want to have a rectangular dance floor that is 5 yards long and 4 yards wide. How much will it cost to rent flooring? *Explain your reasoning.*

If the floor area is ...	Then the cost is ...
less than 100 square feet	\$6.50 per square foot
between 100 and 200 square feet	\$6.25 per square foot

Below are sample solutions to the problem. Read each solution and the comments on the left to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

Find the area of the dance floor. $\text{Area} = \ell w = 5(4) = 20 \text{ yd}^2$.

Then convert this area to square feet. There are $3^2 = 9 \text{ ft}^2$ in 1 yd^2 .

$$20 \text{ yd}^2 \cdot \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = 180 \text{ ft}^2$$

Because 180 ft^2 is between 100 ft^2 and 200 ft^2 , the price of flooring is \$6.25 per square foot. Multiply the price per square foot by the area.

$$\text{Total cost} = \frac{\$6.25}{1 \text{ ft}^2} \cdot 180 \text{ ft}^2 = \$1125$$

It will cost \$1125 to rent flooring.

.....→
The reasoning is correct, and the computations are accurate.

.....→
The answer is correct.

SAMPLE 2: Partial credit solution

The area of the dance floor is $5(4) = 20$ square yards. Convert this area to square feet. There are 3 feet in 1 yard.

$$20 \text{ yd}^2 \cdot \frac{3 \text{ ft}^2}{1 \text{ yd}^2} = 60 \text{ ft}^2$$

The flooring will cost \$6.50 per square foot because 60 ft^2 is less than 100 ft^2 . To find the total cost, multiply the area by the cost per square foot.

$$60 \text{ ft}^2 \cdot \frac{\$6.50}{1 \text{ ft}^2} = \$390$$

It will cost \$390 to rent flooring.

.....→
The reasoning is correct, but an incorrect conversion leads to an incorrect answer.

SAMPLE 3: Partial credit solution

.....→
The computations and the answer are correct, but the reasoning is incomplete.

The area of the room is 180 ft^2 , so the flooring price is \$6.25. The total cost is $180 \cdot 6.25 = \$1125$.

It will cost \$1125 to rent flooring.

SAMPLE 4: No credit solution

.....→
The student's reasoning is incorrect, and the answer is incorrect.

Floor area = $4 \times 5 = 20$.

Cost = $20 \times \$650 = \$13,000$.

It will cost \$13,000 to rent flooring.

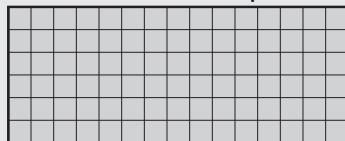
PRACTICE Apply the Scoring Rubric

Use the rubric to score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. Explain your reasoning.

PROBLEM You have 450 daffodil bulbs. You divide a 5 yard by 2 yard rectangular garden into 1 foot by 1 foot squares. You want to plant the same number of bulbs in each square. How many bulbs should you plant in each square? *Explain* your reasoning.

1. First find the area of the plot in square feet. There are 3 feet in 1 yard, so the length is $5(3) = 15$ feet, and the width is $2(3) = 6$ feet. The area is $15(6) = 90$ square feet. The garden plot can be divided into 90 squares with side length 1 foot. Divide 450 by 90 to get 5 bulbs in each square.
2. The area of the garden plot is $5(2) = 10$ square yards. There are 3 feet in 1 yard, so you can multiply 10 square yards by 3 to get an area of 30 square feet. You can divide the garden plot into 30 squares. To find how many bulbs per square, divide 450 bulbs by 30 to get 15 bulbs.
3. Divide 450 by the area of the plot: $450 \text{ bulbs} \div 10 \text{ yards} = 45 \text{ bulbs}$. You should plant 45 bulbs in each square.
4. Multiply the length and width by 3 feet to convert yards to feet. The area is $15 \text{ ft} \times 6 \text{ ft} = 90 \text{ ft}^2$. Divide the garden into 90 squares.

Diagram of garden plot



2 yd = 6 ft

5 yd = 15 ft

Answers

1. Full Credit; the reasoning is correct and the computations are accurate.
2. Partial Credit; the reasoning is correct, but an incorrect conversion leads to an incorrect solution.
3. No Credit; the reasoning is incorrect and the solution is incorrect.
4. Partial Credit; the reasoning is correct, but incomplete.

Answers

1. \$486; the area is 243 square feet, so 243 must be multiplied by \$2.

2. $(-0.5, -1.5)$; 2.9 km; use the Midpoint Formula to find the coordinates of H and then the Distance Formula to find the distance between your house and the library.

3. 4158 gal; the surface area of the pool is 236.25 square feet, multiply 236.25 by 17.6 to find the number of gallons evaporated.

4. About \$140; the area of the pool is about 35 square yards, multiply 35 by \$4 to find the cost.

5. About \$1.88; $x + 5 = 4x - 7$, so $x = 4$. Then the pentagon has 9 in. sides and perimeter = 45 in.; $\frac{\$1.50}{1 \text{ yd}} \cdot 45 \text{ in.} \cdot \frac{1 \text{ yd}}{36 \text{ in.}} \approx \1.88 .

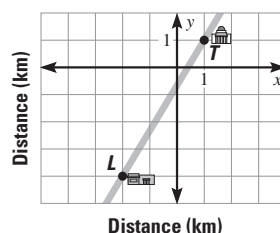
6. 120° ; find the measure of angle B to be 60° , then subtract the measure of angle B from 180° .

7. About 2.2 km; the distance when driving through Baxton is 8 kilometers; the direct distance is about 5.8 kilometers, so their difference is about 2.2 kilometers.

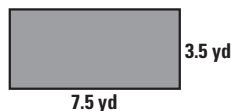
8. 12 pairs; using the same unit, divide the length of wire by the circumference of one earring to find the number of earrings that can be made. Divide that by 2 to find the number of pairs that can be made.

SHORT RESPONSE

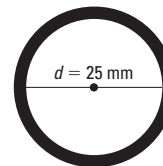
- It costs \$2 per square foot to refinish a hardwood floor if the area is less than 300 square feet, and \$1.75 per square foot if the area is greater than or equal to 300 square feet. How much does it cost to refinish a rectangular floor that is 6 yards long and 4.5 yards wide? *Explain* your reasoning.
- As shown below, the library (point L) and the Town Hall (point T) are on the same straight road. Your house is on the same road, halfway between the library and the Town Hall. Let point H mark the location of your house. Find the coordinates of H and the approximate distance between the library and your house. *Explain* your reasoning.



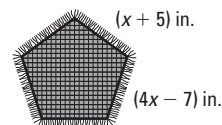
- The water in a swimming pool evaporates over time if the pool is not covered. In one year, a swimming pool can lose about 17.6 gallons of water for every square foot of water that is exposed to air. About how much water would evaporate in one year from the surface of the water in the pool shown? *Explain* your reasoning.



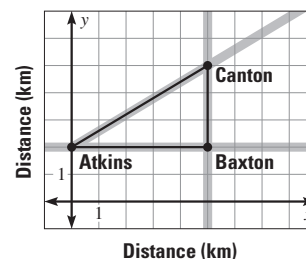
- A company is designing a cover for a circular swimming pool. The diameter of the pool is 20 feet. The material for the cover costs \$4 per square yard. About how much will it cost the company to make the pool cover? *Explain* your reasoning.



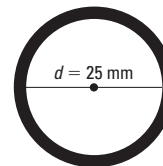
- You are making a mat with a fringed border. The mat is shaped like a regular pentagon, as shown below. Fringe costs \$1.50 per yard. How much will the fringe for the mat cost? *Explain* your reasoning.



- Angles A and B are complementary angles, $m\angle A = (2x - 4)^\circ$, and $m\angle B = (4x - 8)^\circ$. Find the measure of the supplement of $\angle B$. *Explain* your reasoning.
- As shown on the map, you have two ways to drive from Atkins to Canton. You can either drive through Baxton, or you can drive directly from Atkins to Canton. About how much shorter is the trip from Atkins to Canton if you do not go through Baxton? *Explain* your reasoning.



- A jeweler is making pairs of gold earrings. For each earring, the jeweler will make a circular hoop like the one shown below. The jeweler has 2 meters of gold wire. How many pairs of gold hoops can the jeweler make? *Justify* your reasoning.

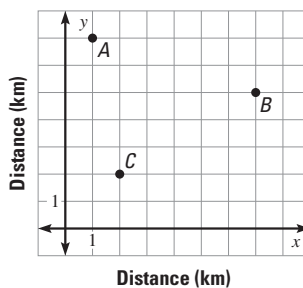


MULTIPLE CHOICE

9. The midpoint of \overline{AB} is $M(4, -2)$. One endpoint is $A(-2, 6)$. What is the length of \overline{AB} ?
- (A) 5 units
(B) 10 units
(C) 20 units
(D) 28 units
10. The perimeter of a rectangle is 85 feet. The length of the rectangle is 4 feet more than its width. Which equation can be used to find the width w of the rectangle?
- (A) $85 = 2(w + 4)$
(B) $85 = 2w + 2(w - 4)$
(C) $85 = 2(2w + 4)$
(D) $85 = w(w + 4)$

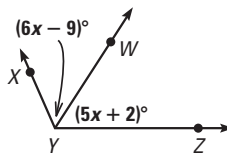
EXTENDED RESPONSE

14. The athletic director at a college wants to build an indoor playing field. The playing field will be twice as long as it is wide. Artificial turf costs \$4 per square foot. The director has \$50,000 to spend on artificial turf.
- What is the largest area that the director can afford to cover with artificial turf? *Explain.*
 - Find the approximate length and width of the field to the nearest foot.
15. An artist uses black ink to draw the outlines of 30 circles and 25 squares, and red ink to fill in the area of each circle and square. The diameter of each circle is 1 inch, and the side length of each square is 1 inch. Which group of drawings uses more black ink, the *circles* or the *squares*? Which group of drawings uses more red ink? *Explain.*
16. Points A and C represent the positions of two boats in a large lake. Point B represents the position of a fixed buoy.
- Find the distance from each boat to the buoy.
 - The boat at point A travels toward the buoy in a straight line at a rate of 5 kilometers per hour. The boat at point C travels to the buoy at a rate of 5.2 kilometers per hour. Which boat reaches the buoy first? *Explain.*

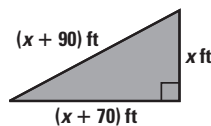


GRIDDED ANSWER

11. In the diagram, \overrightarrow{YW} bisects $\angle XYZ$. Find $m\angle XYZ$ in degrees.



12. Angles A and B are complements, and the measure of $\angle A$ is 8 times the measure of $\angle B$. Find the measure (in degrees) of the supplement of $\angle A$.
13. The perimeter of the triangle shown is 400 feet. Find its area in square feet.



9. C

10. C

11. 114°

12. 100°

13. 6000 ft^2

14a. $12,500 \text{ ft}^2$; divide 50,000 by 4.

14b. 158 ft, 79 ft

15. Square; square; the perimeter of 1 square is 4 inches and there are 25 squares, so there are 100 inches for the squares. The circumference of 1 circle is about 3.14 inches and there are 30 circles, so there are about 94.2 inches for the circles; the area of 1 square is 1 square inch and there are 25 squares, so there are 25 square inches for the squares. The area of 1 circle is about 0.785 square inch and there are 30 circles, so there are about 23.6 square inches for the circles.

16a. boat at point A: $2\sqrt{10} \approx 6.3 \text{ km}$,
boat at point C: $\sqrt{34} \approx 5.8 \text{ km}$

16b. Boat C; boat A takes about 1.3 hours to get to the buoy, and boat C takes about 1.1 hours to get to the buoy.